

CMSC210 (Fall 2003) Discrete Structures

List of Definitions, Notations, and Concepts

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Note: The section and page numbers refer to the text. You may also want to write down pointers to other course materials (lecture slides, exercises, etc.).

1. Sets

<i>set, member/element</i> [definition]	1.2.1
<i>membership</i> relation [definition]: e.g., $b \in A, b \notin A$	1.2.1
list notation [description]: e.g., $\{a, b\}$	1.2.1
predicate notation [description]: e.g., $\{x \mid \dots\}$	1.2.1
<i>singleton (set)</i> [definition]	
<i>empty set</i> [definition]: ' \emptyset '	1.2.1
<i>set identity (= the same set)</i> [definition]: '='	1.2.1
<i>cardinality</i> [definition]: e.g., $ A $ or $\#(A)$	1.2.3
<i>subset</i> relation [definition]: e.g., $A \subseteq B$, Note: $A \subset B$ in the text	1.2.1
<i>proper subset</i> relation [definition]	1.2.1
<i>power set</i> [definition]: e.g., $P(A)$ or $\wp(A)$, Note: $\text{power}(A)$ in the text	1.2.1
Venn diagram [description]	1.2.1
set operations [description]	1.2.2
<i>union</i> [definition]: e.g., $A \cup B$, also for multiple sets, $\bigcup_{i=1}^n A_i$	1.2.2
<i>intersection</i> [definition]: e.g., $A \cap B$, also for multiple sets, $\bigcap_{i=1}^n A_i$	1.2.2
<i>complement</i> [definition]: e.g., A' or \bar{A}	1.2.2
<i>difference</i> [definition]: e.g., $A - B$	1.2.2
universal set (= universe) [description]: U	1.2.2
<i>disjoint</i> [definition]: Sets A and B are <i>disjoint</i> if $A \cap B = \emptyset$	1.2.3

2. Relations

<i>ordered pair</i> [definition]: e.g., (a, b) or $\langle a, b \rangle$	1.3.1
<i>ordered n-tuple</i> [definition]: e.g., (a_1, a_2, \dots, a_n) or $\langle a_1, a_2, \dots, a_n \rangle$	1.3.1
<i>Cartesian product</i> [definition]: e.g., $A \times B$	1.3.1
<i>type</i> [description]: e.g., $R: A \times B$ for a relation R between sets A and B	n/a
<i>binary relation</i> [definition]: e.g., $(a, b) \in R, aRb$, or Rab , also $R \subseteq A \times B$	1.3.4
<i>n-ary/n-place relation</i> [definition]	1.3.4
<i>unary</i> [description]	1.3.4
<i>ternary</i> [description]: 3-ary	n/a
<i>inverse</i> [definition]: e.g., $R^{-1}, R^{-1} = \{(x, y) \mid (y, x) \in R\}$	n/a
properties of relations: <i>reflexive, irreflexive, symmetric, antisymmetric, transitive, intransitive</i> [definition]	4.1
<i>transitive closure</i> of a relation R [definition]: the smallest transitive relation that contains R	4.1.2
<i>X closure</i> of a relation R [definition]: the smallest X relation that contains R	4.1.2
<i>equivalence relation</i> [definition]	4.2.1
<i>equivalence class</i> [definition]: e.g., $[x], [[x]]$, or more explicitly (for relation R), $[x]_R$	4.2.2
<i>partition</i> [definition]	4.2.3

3. Functions

<i>function (= mapping)</i> [definition]:	2.1.1
<i>n-ary/n-place</i> function [description]: function with more than one arguments	2.1.1
<i>type</i> [description]: e.g., $f: A \rightarrow B$ for a function f between sets A and B	n/a

<i>inverse</i> [definition]: e.g., f^{-1}	2.3.2
<i>onto/surjective, one-to-one/injective</i> [definition]	2.3.1
<i>bijective</i> [definition]	2.3.2
<i>composition</i> [definition]: e.g., $g \circ f$ or gf	2.2.1
recursive function [description]: function that is defined in terms of itself	3.2
recursion [description]: instance/use of recursive function, definition, etc.	n/a

Example functions

<i>factorial</i> [definition]: $n! = n \times (n - 1) \times (n - 2) \times \dots \times 2 \times 1$	p. 150
<i>floor</i> (of x) [definition]: greatest integer $\leq x$, $\lfloor x \rfloor$	2.1.2
<i>ceiling</i> (of x) [definition]: smallest integer $\geq x$, $\lceil x \rceil$	2.1.2
<i>remainder</i> [definition]: remainder when m is divided by n , $m \bmod n$	2.1.2

4. Structures

General Ideas

<i>structure</i> [definition]: n -tuple involving sets, relations, and functions	n/a
<i>closure</i> (of operation) [definition] for a binary operation $f: A \times A \rightarrow B$: the range of $f = A$	n/a
operation [description]: concept that is represented by a function	n/a
properties of operations for $\bullet: A \times A \rightarrow B$	
<i>associative</i> [definition]: for any x, y, z , $(x \bullet y) \bullet z = x \bullet (y \bullet z)$	n/a
<i>commutative</i> [definition]: for any x, y, z , $x \bullet y = y \bullet x$	n/a
<i>distributive</i> [definition]	p. 573
De Morgan's law [description]	1.2.2, 6.2.2, 10.2.1
special elements: <i>identity element, inverse</i> [definition]	10.1.3
<i>isomorphism</i> [definition]	10.5.4

Strings

<i>semigroup, monoid, group</i> [definition]	10.1.3
<i>string, empty string</i> (= null string, ϵ), <i>concatenation</i> [definition]: Note: empty string as Λ in the text	1.3.3

Orderings

<i>poset</i> (= <i>partially ordered set</i>) [definition]: e.g., (A, \leq)	4.3.1
<i>Hasse diagram, comparable</i> [definition]	4.3.1
<i>least, greatest</i> [definition]	4.3.1
<i>total/linear order</i> (= <i>chain</i>), <i>well-ordered set</i> [definition]	4.3.1
<i>Boolean algebra</i> [definition]	10.2

Graphs and Trees

<i>graph</i> (= <i>undirected graph</i>) [definition]: (V, E) where V is a non-empty set and E is an irreflexive, symmetric relation on V	1.4.1
<i>directed graph</i> (= <i>digraph</i>) [definition]: graph where E is not necessarily symmetric	1.4.1
<i>vertex</i> (= <i>node</i>) [definition]: element of V	1.4.1
<i>edge</i> [definition]: element of E	1.4.1
<i>path</i> from a to b [definition]: if the transitive closure of E contains (a, b)	1.4.2
<i>connected</i> [definition]: if there is a path from every vertex to any other one	1.4.2
<i>cycle</i> [definition]: a path from a vertex to itself	1.4.2
<i>acyclic</i> [definition]: with no cycles	1.4.2
<i>tree</i> [definition]: connected, acyclic, undirected graph	1.4.4
root [description]: a designated vertex of a tree	1.4.4
<i>ancestor</i> of v [definition]: u such that there is a path between v and the root through u	1.4.4
<i>descendant</i> of v [definition]: u such that v is an ancestor of u	1.4.4
<i>parent</i> (= <i>mother</i>) of v [definition]: an ancestor of v , u , such that $(u, v) \in E$	1.4.4

<i>child</i> (= <i>daughter</i>) of v [definition]: u such that v is a parent of u	1.4.4
<i>leaf</i> [definition]: a vertex with no children	1.4.4
<i>height</i> [definition]: the maximum length between a leaf and the root	1.4.4

Languages and Automata

language [description]	1.3.3, 3.3.3
<i>regular language</i> (= <i>regular set</i>) [definition]	n/a
automaton [description]	n/a
<i>finite-state automaton</i> (= <i>finite-state machine</i>) [definition]	n/a
grammar [description]	3.3
<i>regular expression</i> [definition]	
$X^n = \underbrace{X \dots X}_n$	n/a

Counting

<i>entropy</i> [definition]: informally, the amount of information; mathematically, $-\log_2 p$ (for uniform distribution with the probability p) or $-\sum_{i=1}^n p_i \log_2 p_i$ (for a more general distribution)	n/a
probability [description]:	
sample space, random variable [definition]	5.3.3
frequentist vs. Bayesian [description]	n/a
multiplication principle [description]: for finite sets, $ A \times B = A \times B $	n/a
addition principle [description]: for finite disjoint sets, $ A \cup B = A + B $	n/a
<i>permutation</i> [definition]: number of r -permutation of an n -element set, $P(n, r)$ or ${}_n P_r$	5.3.1
<i>combination</i> [definition]: number of r -combination of an n -element set, $C(n, r)$ or ${}_n C_r$	5.3.2
pigeon hole principle [description]: for finite sets, if $ A > B $, there is no injection from A to B	2.3.3

5. Logic

General Ideas

object language [description]: language of a that is the target of study	n/a
meta-language [description]: language used to discuss another language (object language)	n/a
syntax, semantics [description]	6.2.1
<i>interpretation</i> [definition]: specification of the meaning of logical symbols	7.1.3
logical formula (= formula) [description]: a complete sequence of logical symbols	n/a
<i>satisfaction</i> [definition]: relation between structures and logical formulas, e.g., $S \models \phi$	n/a
<i>logical consequence/implication</i> [definition]: a set of formulas follow from another set of formula	n/a
<i>logical equivalence</i> [definition]	n/a
<i>valid</i> [definition]: logical consequence of no formula	7.1.4
<i>theory</i> [definition]: a set of formulas that are closed under logical consequence	n/a
proof system [description]	6.4
<i>axiom, rule of inference, proof, theorem</i> [definition]	6.3.1-6.3.2
<i>modus ponens</i> [definition]	6.3.1
<i>consistent, inconsistent</i> [definition]: there is a truth value assignments that makes all statements true	6.3.2

Propositional Logic

propositional logic (= statement logic) [description]	6.2
proposition (= statement) [description]	6.2
truth value, true, false [description]	6.2
propositional variable [description]	(6.2)
connective [description]	6.2
Language/syntax	
<i>negation</i> [definition]: e.g., $\neg p$, $\sim p$, or \bar{p}	6.2

<i>conjunction</i> [definition]: e.g., $p \wedge q$ or $p \& q$	6.2
<i>disjunction</i> [definition]: e.g., $p \vee q$	6.2
<i>conditional</i> [definition]: e.g., $p \rightarrow q$ or $p \supset q$	6.2
<i>biconditional</i> (= <i>if and only if, iff</i>) [definition]: e.g., $p \leftrightarrow q$	n/a
<i>exclusive or</i> [definition]: $p \vee q$ but not $p \wedge q$, e.g., $p \oplus q$	n/a
<i>well-formed formula</i> (= <i>wff</i>) [definition]	6.2.1
Semantics	
<i>interpretation</i> [definition]: the semantics of logical connectives	n/a
<i>assignment</i> (of propositional variables) [definition]: function that maps the set of propositional variables to $\{T, F\}$	n/a
truth table [description]	6.2.1
<i>tautology, contradiction</i> [definition]	6.2.1
<i>contrapositive</i> [definition]: $\neg q \rightarrow \neg p$ in relation to $p \rightarrow q$	1.1
First-Order Logic	
first-order logic (= FOL, predicate logic) [description]	7.1
Language/syntax	
<i>individual constant, individual variable</i> [definition]	7.1.2
<i>predicate symbol/constant, function symbol/constant</i> [definition]	7.1.2
<i>term, predicate</i> [definition]	7.1.2
universal quantifier [description]: ‘ \forall ’	7.1.1
existential quantifier [description]: ‘ \exists ’	7.1.1
Semantics	
<i>interpretation</i> [definition]: the semantics of logical connectives	7.1.3
<i>assignment</i> (of individual variables) [definition]: function that maps the set of individual variables to the set of individual elements	7.1.3
<i>evaluation</i> (= <i>semantic value</i>) [definition]: identifying the individual that corresponds to a term, e.g., $[[x]]$	n/a
Manner of arguments	
deduction [general meaning]: general \rightarrow specific	n/a
deduction [in logic]: derivation from general rules (as in ‘natural deduction’)	n/a
induction [general meaning]: specific \rightarrow general	n/a
induction [in logic/math]: e.g., mathematical induction	4.4.1
inductive definition (= recursive definition) [description]	3.1
mathematical induction [description]	4.4.1
recursive/inductive definition [description]: definition that refers itself	3.1
base case/basis [description]: case not involving a recursion	3.1
induction (step) [description]: case involving a recursion	3.1
exclusion clause [description]: statement to exclude unintended elements (for recursive definition of a set)	3.1
proof styles	
direct proof [description]	1.1
indirect proof [description]	1.1
proof by contradiction (= <i>reductio ad absurdum</i>) [description]	1.1
corollary [description]: theorem easily obtainable from another theorem	n/a
lemma [description]: theorem used to prove another theorem	n/a

<End>