CMSC210 (Fall 2003) Discrete Structures
List of Definitions, Notations, and Concepts

July 28, 2003

Note: The section and page numbers refer to the text. You may also want to write down pointers to other course materials (lecture slides, exercises, etc.).

1. Sets

set, *member*/*element* [definition] 1.2.1
membership relation [definition]: e.g., $b \in A$, $b \notin A$ 1.2.1
list notation [description]: e.g., $\{a, b\}$ 1.2.1
predicate notation [description]: e.g., $\{x \mid ...\}$ 1.2.1
singleton (*set*) [definition]
empty set [definition]: '$\varnothing$' 1.2.1
set identity (= *the same set*) [definition]: '=' 1.2.1
cardinality [definition]: e.g., $|A|$ or $\#(A)$ 1.2.3
subset relation [definition]: e.g., $A \subseteq B$, Note: $A \subset B$ in the text 1.2.1
    proper subset relation [definition] 1.2.1
power set [definition]: e.g., $P(A)$ or $\wp(A)$, Note: power(A) in the text 1.2.1
Venn diagram [description] 1.2.1
set operations [description] 1.2.2
    union [definition]: e.g., $A \cup B$, also for multiple sets, $\bigcup_{i=1}^{n} A_i$ 1.2.2
    intersection [definition]: e.g., $A \cap B$, also for multiple sets, $\bigcap_{i=1}^{n} A_i$ 1.2.2
    complement [definition]: e.g., A' or $\overline{A}$ 1.2.2
    difference [definition]: e.g., $A - B$ 1.2.2
    universal set (= universe) [description]: U 1.2.2
disjoint [definition]: Sets A and B are *disjoint* if $A \cap B = \varnothing$ 1.2.3

2. Relations

ordered pair [definition]: e.g., (a, b) or $\langle a, b \rangle$ 1.3.1
    ordered n-tuple [definition]: e.g., $(a_1, a_2, ..., a_n)$ or $\langle a_1, a_2, ..., a_n \rangle$ 1.3.1
Cartesian product [definition]: e.g., $A \times B$ 1.3.1
type [description]: e.g., R: $A \times B$ for a relation R between sets A and B n/a
binary relation [definition]: e.g., $(a, b) \in R$, aRb, or Rab, also $R \subseteq A \times B$ 1.3.4
    n-ary/*n-place relation* [definition] 1.3.4
    unary [description] 1.3.4
    ternary [description]: 3-ary n/a
inverse [definition]: e.g., R^{-1}, $R^{-1} = \{(x, y) \mid (y, x) \in R\}$ n/a
properties of relations: *reflexive*, *irreflexive*, *symmetric*, *antisymmetric*, *transitive*, *intransitive* [definition] 4.1
    transitive closure of a relation R [definition]: the smallest transitive relation that contains R 4.1.2
    X closure of a relation R [definition]: the smallest *X* relation that contains R 4.1.2
equivalence relation [definition] 4.2.1
equivalence class [definition]: e.g., $[x]$, $[[x]]$, or more explicitly (for relation R), $[x]_R$ 4.2.2
partition [definition] 4.2.3

3. Functions

function (= *mapping*) [definition]: 2.1.1
    n-ary/*n*-place function [description]: function with more than one arguments 2.1.1
type [description]: e.g., f: $A \rightarrow B$ for a function f between sets A and B n/a

inverse [definition]: e.g., f^{-1} 2.3.2
onto/*surjective*, *one-to-one*/*injective* [definition] 2.3.1
bijective [definition] 2.3.2
composition [definition]: e.g., $g \circ f$ or gf 2.2.1
recursive function [description]: function that is defined in terms of itself 3.2
recursion [description]: instance/use of recursive function, definition, etc. n/a

Example functions

factorial [definition]: $n! = n \times (n-1) \times (n-2) \times ... \times 2 \times 1$ p. 150
floor (of x) [definition]: greatest integer $\leq x$, $\lfloor x \rfloor$ 2.1.2
ceiling (of x) [definition]: smallest integer $\geq x$, $\lceil x \rceil$ 2.1.2
remainder [definition]: remainder when m is divided by n, m mod n 2.1.2

4. Structures

General Ideas

structure [definition]: n-tuple involving sets, relations, and functions n/a
closure (of operation) [definition] for a binary operation $f: A \times A \rightarrow B$: the range of $f = A$ n/a
operation [description]: concept that is represented by a function n/a
properties of operations for $\bullet: A \times A \rightarrow B$
associative [definition]: for any x, y, z, $(x \bullet y) \bullet z = x \bullet (y \bullet z)$ n/a
commutative [definition]: for any x, y, z, $x \bullet y = y \bullet x$ n/a
distributive [definition] p. 573
De Morgan's law [description] 1.2.2, 6.2.2, 10.2.1
special elements: *identity element*, *inverse* [definition] 10.1.3
isomorphism [definition] 10.5.4

Strings

semigroup, *monoid*, *group* [definition] 10.1.3
string, *empty string* (= *null string*, ε), *concatenation* [definition]: Note: empty string as Λ in the text 1.3.3

Orderings

poset (= *partially ordered set*) [definition]: e.g., $(A, \leq)$ 4.3.1
Hasse diagram, *comparable* [definition] 4.3.1
least, *greatest* [definition] 4.3.1
total/*linear order* (= *chain*), *well-ordered* set [definition] 4.3.1
Boolean algebra [definition] 10.2

Graphs and Trees

graph (= *undirected graph*) [definition]: (V, E) where V is a non-empty set and E is an irreflexive, symmetric relation on V 1.4.1
directed graph (= *digraph*) [definition]: graph where E is not necessarily symmetric 1.4.1
vertex (= *node*) [definition]: element of V 1.4.1
edge [definition]: element of E 1.4.1
path from a to b [definition]: if the transitive closure of E contains (a, b) 1.4.2
connected [definition]: if there is a path from every vertex to any other one 1.4.2
cycle [definition]: a path from a vertex to itself 1.4.2
acyclic [definition]: with no cycles 1.4.2
tree [definition]: connected, acyclic, undirected graph 1.4.4
root [description]: a designated vertex of a tree 1.4.4
ancestor of v [definition]: u such that there is a path between v and the root through u 1.4.4
descendant of v [definition]: u such that v is an ancestor of u 1.4.4
parent (= *mother*) of v [definition]: an ancestor of v, u, such that $(u, v) \in E$ 1.4.4

child (= *daughter*) of *v* [definition]: *u* such that *v* is a parent of *u* 1.4.4
leaf [definition]: a vertex with no children 1.4.4
height [definition]: the maximum length between a leaf and the root 1.4.4

Languages and Automata

language [description] 1.3.3, 3.3.3
regular language (= *regular set*) [definition] n/a
automaton [description] n/a
finite-state automaton (= *finite-state machine*) [definition] n/a
grammar [description] 3.3
regular expression [definition]
$X^n = \underbrace{X \ldots X}_{n}$ n/a

Counting

entropy [definition]: informally, the amount of information; mathematically, $-\log_2 p$ (for uniform distribution with the probability p) or $-\sum_{i=1}^{n} p_i \log_2 p_i$ (for a more general distribution) n/a
probability [description]:
sample space, random variable [definition] 5.3.3
frequentist vs. Bayesian [description] n/a
multiplication principle [description]: for finite sets, $|A \times B| = |A| \times |B|$ n/a
addition principle [description]: for finite disjoint sets, $|A \cup B| = |A| + |B|$ n/a
permutation [definition]: number of *r*-permutation of an *n*-element set, $P(n, r)$ or ${}_nP_r$ 5.3.1
combination [definition]: number of *r*-combination of an *n*-element set, $C(n, r)$ or ${}_nC_r$ 5.3.2
pigeon hole principle [description]: for finite sets, if $|A| > |B|$, there is no injection from *A* to *B* 2.3.3

5. Logic

General Ideas

object language [description]: language of a that is the target of study n/a
meta-language [description]: language used to discuss another language (object language) n/a
syntax, semantics [description] 6.2.1
interpretation [definition]: specification of the meaning of logical symbols 7.1.3
logical formula (= formula) [description]: a complete sequence of logical symbols n/a
satisfaction [definition]: relation between structures and logical formulas, e.g., $S \models \varphi$ n/a
logical consequence/*implication* [definition]: a set of formulas follow from another set of formula n/a
logical equivalence [definition] n/a
valid [definition]: logical consequence of no formula 7.1.4
theory [definition]: a set of formulas that are closed under logical consequence n/a
proof system [description] 6.4
axiom, rule of inference, proof, theorem [definition] 6.3.1-6.3.2
modus ponens [definition] 6.3.1
consistent, inconsistent [definition]: there is a truth value assignments that makes all statements true 6.3.2

Propositional Logic

propositional logic (= statement logic) [description] 6.2
proposition (= statement) [description] 6.2
truth value, true, false [description] 6.2
propositional variable [description] (6.2)
connective [description] 6.2
Language/syntax
negation [definition]: e.g., $\neg p$, $\sim p$, or $\overline{p}$ 6.2

conjunction [definition]: e.g., $p \wedge q$ or $p \,\&\, q$ 6.2
disjunction [definition]: e.g., $p \vee q$ 6.2
conditional [definition]: e.g., $p \rightarrow q$ or $p \supset q$ 6.2
biconditional (= *if and only if*, *iff*) [definition]: e.g., $p \leftrightarrow q$ n/a
exclusive or [definition]: $p \vee q$ but not $p \wedge q$, e.g., $p \oplus q$ n/a
well-formed formula (= *wff*) [definition] 6.2.1

Semantics

interpretation [definition]: the semantics of logical connectives n/a
assignment (of propositional variables) [definition]: function that maps the set of propositional variables to {*T*, *F*} n/a
truth table [description] 6.2.1

tautology, *contradiction* [definition] 6.2.1
contrapositive [definition]: $\neg q \rightarrow \neg p$ in relation to $p \rightarrow q$ 1.1

First-Order Logic

first-order logic (= FOL, predicate logic) [description] 7.1

Language/syntax

individual constant, *individual variable* [definition] 7.1.2
predicate symbol/constant, *function symbol/constant* [definition] 7.1.2
term, *predicate* [definition] 7.1.2
universal quantifier [description]: '$\forall$' 7.1.1
existential quantifier [description]: '$\exists$' 7.1.1

Semantics

interpretation [definition]: the semantics of logical connectives 7.1.3
assignment (of individual variables) [definition]: function that maps the set of individual variables to the set of individual elements 7.1.3
evaluation (= *semantic value*) [definition]: identifying the individual that corresponds to a term, e.g., [[*x*]] n/a

Manner of arguments

deduction [general meaning]: general → specific n/a
deduction [in logic]: derivation from general rules (as in 'natural deduction') n/a
induction [general meaning]: specific → general n/a
induction [in logic/math]: e.g., mathematical induction 4.4.1
inductive definition (= recursive definition) [description] 3.1
mathematical induction [description] 4.4.1
recursive/inductive definition [description]: definition that refers itself 3.1
base case/basis [description]: case not involving a recursion 3.1
induction (step) [description]: case involving a recursion 3.1
exclusion clause [description]: statement to exclude unintended elements (for recursive definition of a set) 3.1
proof styles
direct proof [description] 1.1
indirect proof [description] 1.1
proof by contradiction (= reductio ad absurdum) [description] 1.1
corollary [description]: theorem easily obtainable from another theorem n/a
lemma [description]: theorem used to prove another theorem n/a

<End>