Unit A1: Mathematical Modeling, 8/29/03

Exercise 1: Objects in a Room

Consider the following conditions about objects in a room [remember the initial survey?]:

1. An object must have another object on top of it.
2. An object cannot be on top of itself.
3. If an object *X* is on top of another object *Y*, *Y* cannot be on top of *X*.

Then, consider the following five scenarios and respond to the associated questions (and one more question). Note that we can view the conditions as 'logic' and the scenarios as 'structures'. Thus, we examine whether logic *specifies* a particular structure.

A. **Scenario**: The room does not have a ceiling. There is an object (call it Object #1) on the floor. There is another object (call it Object #2) on top of Object #1. There is yet another object (call it Object #3) on top of Object #2. More objects are stacked in this manner without limit.
Question: Examine whether this scenario is consistent with the three conditions. In other words, is any of the three conditions violated?

Hint: Visualize this and the following scenarios by drawing pictures.

B. **Scenario**: There are three objects.
Question: Would it be possible to satisfy all of the three conditions? Explain.

Hint: Observe M.C. Escher's *Ascending and Descending* (if not familiar, do a web search).

C. **Scenario**: There are two objects.
Question: Would it be possible to satisfy all of the three conditions? Explain.

D. **Scenario**: There is one object.
Question: Would it be possible to satisfy all of the three conditions? Explain.

E. **Scenario**: There are no objects.
Question: Would it be possible to satisfy all of the three conditions? Explain.

F. Are there any other possibility? Explain.

Exercise 2: Professionals

Let us consider a real-world (?) situation involving professionals. The conditions (logic) are as follows:

1. Everyone is mad.
2. There is at least one doctor.

3. There are at least two lawyers.
4. Doctors are not lawyers.
5. Lawyers sue everyone.
6. Doctors sue back if they are sued.
7. There is an individual who does not sue.

We now find out a possible scenario (structure) consisting of the following components:

- List of involved people (give arbitrary names to distinguish them)
- List of lawyers, doctors, and mad people
- Information about who sues whom

Find a scenario (structure) that is consistent with all the conditions and involves the smallest number of individuals.

Hint: Again, draw a picture.

Exercise 3: Create Your Own Logic-Structure Connection

We have seen a few examples of logic-structure connection ('North Pole' in class, H1 and H2 above). In this exercise, you will create your own such example. First, find a topic of your interest and imagine a certain situation/scenario (structure in your mind). Do not write down the scenario on the exercise sheet; keep it in your mind. Then, write down logical statements (in a way similar to the examples) so that your scenario is consistent with the logical statements. Also try to make the logical statements 'precise', 'correct', and 'concise' (review lecture slides on these terms).

Note: In case you find this difficult, explain why it is difficult and what you will need to know to do this exercise.

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