

Unit A3: Structures, 9/9/03

Exercise 1: Addition vs. Multiplication

In this exercise, we compare structures involving (i) a set of numbers and (ii) *either* addition *or* multiplication as the only operation on the numbers. We do this comparison for the following two different cases: (A) the set of numbers is limited to positive integers and (B) the set contains all real numbers.

- A. As the set of objects, consider only positive integers, i.e., 1, 2, 3, ... (i.e., no greatest number). Then, we consider two structures **Addition_{Int+}** and **Multiplication_{Int+}**: **Addition_{Int+}** is a structure consisting of the set of positive integers and the usual addition operation '+', and **Multiplication_{Int+}** is a structure consisting of the set of positive integers and the usual multiplication operation, '×'. Compare these structures with respect to the property “existence of an identity element” and evaluate whether or not they are similar.

Note: An identity element x must satisfy $x + y = y + x = y$ for any y in the set designated for the structure under consideration. That is, for this part, A, you must consider all positive integers but must exclude numbers such as 0, -1, and 0.5.

- B. As the set of objects, consider all real numbers. Then, we consider two structures **Addition_{Real}** and **Multiplication_{Real}**: **Addition_{Real}** is a structure consisting of the set of real numbers and the usual addition operation '+', and **Multiplication_{Real}** is a structure consisting of the set of real numbers and the usual multiplication operation, '×'. Compare these structures with respect to the property “existence of an identity element” and evaluate whether or not they are similar.

Exercise 2: Drug Trafficking

Drug trafficking involves a variety of players. In this exercise, we will represent a hypothetical situation of your choice as a structure. You must decide on the level of details and complexity. Since this is an exercise in Discrete Math, not in Social Science, you need to reflect *some* aspect of drug trafficking. It does not need to be complete. [Optional (but definitely not required) reading: The Social Impact of Drug Abuse from United Nations Office on Drugs and Crime at http://www.unodc.org/pdf/technical_series_1995-03-01_1.pdf (clickable in the on-line version)]

- A. Represent your situation as a structure, i.e., as a combination of sets, relations, and/or functions. Give general description for each involved component.
- B. Compare the structure you obtained in Question A above with the structures discussed in class. Try to find similar structures. Explain.

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