A4: Logic

Today

- Identify and use the basic components of logic
  - Logical statements
  - Syntax and semantics
  - Logic-structure connection
- Due by the next class
  - Take-home exercise self-evaluation
  - Module A Comprehensive Exercises

Section 1

Logical Statements

$\forall x \exists y \, (r(x) \rightarrow c(x, y))$

- North Pole
  - Reindeer are not Santa Claus.
  - Reindeer must carry someone/something.
  - Santa Claus must be carried by reindeer.
  - Reindeer exists.
- Properties of an operation/function
  - For any x, y, z, $(x + y) + z = x + (y + z)$
  - There is an element x such that $x + y = y + x = y$ for any y.

'Truth Condition'

- Logical statements must be either true or false.
- Examples/Counterexamples
  - $1 + 1 = 2$
  - $1 + 1 = 3$
  - 12345678901234567 is a prime number.
  - There is nothing outside the universe.
  - Is this sentence true?
  - This sentence is false.

Primitive Statements

- Property
  - Bill is honest.
  - George is intelligent.
- Equality
  - Kirk Douglas = Issur Danielovitch
  - I (first person singular) = Meg Ryan

Complex Statements

- Combination
  - George is intelligent and Bill is honest.
  - Michael Keaton ≠ Michael Douglas
  - If George is intelligent, then 1 + 1 = 1.
- Referring to quantity
  - There is at least one student in this room.
  - Every woman kicks some man.

Any limit to complexity?

Limitations

- Impossible to interpret precisely
  - Unknown symbols
    - E.g., Γεοργε ισ ιντελλιγεντ.
  - Variable interpretation
    - E.g., He is elegant.
- The condition that needs to be represented is too complicated.
  - E.g., one's romantic experience

Section Summary

- A variety of conditions can be represented as logical statements.
- There are conditions that cannot be represented as logical statements.
- Logical statements must be understood by the users unambiguously.

Section 2

Syntax and Semantics

? a?? t?? e?s a?? ?? t? ? s??????? ? ...

- Logical statements are understandable because we can *interpret* symbols in a specific way.
- Symbols and their combination ⇒ Syntax (form)
- Interpretation of symbols ⇒ Semantics (meaning)

Examples

- Analyze the syntax *and* semantics
  - Syntax (primitive, complex)
  - Semantics (truth value, context)

1. George is intelligent.
2. Kirk Douglas = Issur Danielovitch
3. (George is intelligent) and (Kirk Douglas ≠ Issur Danielovitch)
4. Everyone loves Raymond.

Group Exercise

- Analyze the syntax *and* semantics
  - Syntax (primitive, complex)
  - Semantics (truth value, context)

1. There is at least one student in this room.
2. If George is intelligent, then 1 + 1 = 1.

Summary: Combination

- "and": true if both are true
- "or": true if at least one is true
- "not": truth value inversion
- "if": slightly complicated [more later]

Summary: Quantity

- "all" / "every" / "any": all the objects of a particular property
  - E.g., Every living form dies.
- "some" / "a/an": *at least one* object of a particular property
  - E.g., Some computer program never terminates.

Section Summary

- Logical statements follows certain syntactic constraints.
- Meaning of a logical statement can be obtained by analyzing each component one by one.

Section 3

Logic-Structure Connection

- Example
  - Logic: There is at least one object.
  - Structure: Any structure that has at least one object

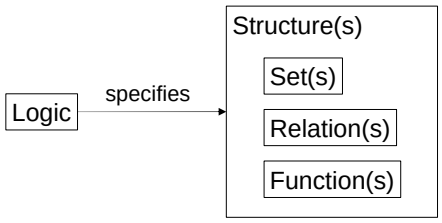
Underlying Mechanism

- Logic
  - Syntax: How to use/combine symbols
  - Semantics: How to interpret symbols
- Structure: Must be consistent with the *semantics* of the logic

Section Summary

- The logic-structure connection is based on the syntax-semantics aspects of logic.
- Logical statements may specify multiple structures.
- In general, more logical statements $\Rightarrow$ fewer structures
- Inconsistent logical statements (e.g., p and not p) $\Rightarrow$ no structures

Module A Summary



Are we done with Discrete Math?

Module A Evaluation Workshop

- Tue., Sep. 16, class time (next class)
- Review the relevant part of the syllabus and the on-line handbook
- Prepare a manila folder or a large envelope
- Complete and bring "Take-Home Exercise Self-Evaluation Form" (distributed in last class) along with exercises
- Exercise A3 will be returned that day. Include it in the pile then.
- Complete and bring "Module A Comprehensive Exercises" (available on-line)
- Group evaluation sessions (open book): 20 min $\times$ 3
- "Comprehensive Exercise Self-Evaluation Forms" will be distributed that day (no need to print in advance)
- Submit all materials at the end of the session

Understood? Can teach?

Summary Exercise

- Explain why logic can be used to convey precise meaning
- Questions/Comments/Suggestions