Review

B1: Extra Exercises

A. Draw a Venn diagram to analyze the general case of involving three sets and formally represent each region.

What about if four sets are involved?

B. Justify: $(A \cap B)' = A' \cup B'$

C. Describe the following in English:

$(A \cap B \cap C)' = A' \cup B' \cup C'$

For your practice

Foreign Policy

[Final Exam Practice, Spring 2003]

- Suppose that you are the foreign minister of a country whose foreign policy is that "an enemy of an enemy is a friend." In a UN session, you must defend this policy.
- Then, you insist that every other country is "either with us or against us."

What kind of tool should we use to convince others?

B2: Relations

Today

- Formally represent relations and use them to analyze phenomena
  - Formal representation of relation
  - Relation type
  - Basic properties of relation
- Take-home exercises
  - Professionals, Electronic Map

getting more abstract

Section 1

Relations in "Foreign Policy"

- "an enemy of an enemy is a friend"
  - "is a friend of"
  - "is an enemy of"
- "either with us or against us"
  - Explain this in terms of the connection between "is a friend of" and "is an enemy of"?
  - Explain this using the combined relation "is an enemy of an enemy of"?

Other Relation Examples

- Barbie dolls
- North pole
- Objects in a room
- Drug trafficking
- Professionals
- Corporate organization
- Computer files

General Properties of Relation

- Relation can be between two/two or more objects.
- Relation
  - Must be between objects from the same set/
  - Can be between objects from different sets.
- In general, the object ordering matters/ does not matter.
- In general, one object can/cannot be related to multiple objects.

Justification/examples?

Formal Representation of Relation

For the case of relation between *two* objects

- Requirements: Must be able to
  - Combine two objects
  - Represent ordering between the objects
  - Include a number of object associations
  - Extend to relations between more than two objects

Use of sets?

Ordered Pair

- A pair of two elements in a *specific order*
  - Notation: (a, b) or $\langle a, b \rangle$
  - Property: $(a, b) = (c, d) \Leftrightarrow a = c$ **and** $b = d$
- Examples
  - $doll_1$ wears $shirt_2 \Rightarrow (doll_1, shirt_2)$
  - l_2 sues $l_2 \Rightarrow (l_2, l_2)$
  - $file_{123}$ is in $folder_{234} \Rightarrow (file_{123}, folder_{234})$
- Note: $\{d_1, s_2\} = \{s_2, d_1\}$, but $(d_1, s_2) \neq (s_2, d_1)$

Binary Relation

- A *set* of ordered pairs that characterize a certain association between two objects
- Example
  - $R = \{(d_1, s_2), (d_1, p_4), (d_2, s_3)\}$ Schematic representation

    name — actual definition
- Membership = The relation holds for the member (pair)
- Notation: $(d_1, s_2) \in R$, d_1Rs_2, $R(d_1, s_2)$

Extension to Multiple Objects

- Extension of ordered pair
- Extension of binary relation

(Ordered) *n*-Tuple

- Extension of ordered pairs to n elements
  - Notation: $(a_1, a_2, ..., a_n)$ or $\langle a_1, a_2, ..., a_n \rangle$
  - Special cases: triple, quadruple, etc.
- Example: $(1, 2, 3) \neq (1, 3, 2) \neq (2, 1, 3)$, etc.

n-ary Relation

- Extension of binary relation to n elements
  - Notation: $(a, b, c) \in R$, $R(a, b, c)$
- Special names
  - Unary: 1-ary relation (= set or 'property')
    - E.g., set of animals = unary relation on objects that have the "animal" property
    - Notation: *Animal(George)*
  - Ternary: 3-ary relation
    - E.g., "United 112 connects PHL and LAX"

Section Summary

To represent a relation

- Connect two or more things ⇒ pair/n-tuple
  - E.g., (a, b) / $(a_1, a_2, ..., a_n)$
- Collect these ⇒ set of pairs/n-tuples
  - E.g.,$\{(d_1, s_2), (d_1, p_4), (d_2, s_3)\}$, $\{(a_1, a_2, ..., a_n), ...\}$

Group Exercise

A. Formally represent the reporting structure in your family as a relation.

B. Formally represent the circle $x^2 + y^2 = 1$ as a relation. In addition, explain whether this relation can be a function. Let us define: **R** = the set of real numbers.

Section 2

Classifying Relation

- Relation involving objects from a single set
  - E.g., "is placed above"
- Relation involving objects from different sets
  - E.g., "*file* is contained in *directory*"

How to provide information about sets (from which objects are taken)?

Not in Text!

Relation Type

- Relation involving objects from a single set
  - E.g., "is placed above"
  - I.e., $(a, b) \in P$ where $a \in O$ and $b \in O$
  - Type: $O \times O$
- Relation involving objects from different sets
  - E.g., "*file* is contained in *directory*"
  - I.e., $(x, y) \in C$ where $x \in F$ and $y \in D$
  - Type: $F \times D$

cf. data type in programming

Cartesian Product

- Cartesian product of sets A and B:
  $\{(x, y) \mid x \in A$ **and** $y \in B\}$
  - Notation: $A \times B$
- Examples
  - $A = \{a, b\}, B = \{1, 2, 3\}$
  - $A \times A = \{(a, a), (a, b), (b, a), (b, b)\}$
  - $A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

2D plane, 3D space?

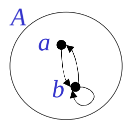
Binary Relation Description

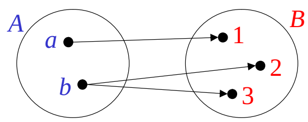
- Equivalent descriptions of binary relation R
  - Associates objects in set A and objects in set B
  - Has type $A \times B$
  - $R \subseteq A \times B$
- Example
  - $A = \{a, b\}, B = \{1, 2, 3\}$
  - $R = \{(a, 1),$ $(a, 2),$ $(a, 3),$ $(b, 1),$ $(b, 2)$ $(b, 3)\}$

$A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

Schematic Representation

- $A = \{a, b\}$, $B = \{1, 2, 3\}$
- $R \subseteq A \times A$
  and $R = \{(a, b), (b, a), (b, b)\}$
- $S \subseteq A \times B$
  and $S = \{(a, 1), (b, 2), (b, 3)\}$





- Identify the type
- Identify set members
- Use arrows

Section Summary

- Type of a relation indicates the source of objects.
- The Cartesian product (e.g., $A \times B$) is used to indicate the type of a relation.

Group Exercise

Recall “Foreign Policy”

- Define: $C = \{1, 2, 3, 4\}$
- Define: $friendOf = \{(1, 2), (2, 1), (2, 3), (3, 2)\}$
- Define: $enemyOf = ?$

1. Types?
2. Would it be reasonable to analyze $enemyOf$ as the complement of $friendOf$? Why?
3. Is an enemy of an enemy of a friend?

Section 3

Properties of Relation

For, e.g., structure classification

- Example relations
  - ‘<’
  - ‘≤’
  - “reports to” (corporate organization)
  - “sits next to” (people in class)
  - “sues” (people)
  - “is networked with” (computers)

Schematic representation

Reflexivity

- For a relation $R \subseteq A \times A$
- Reflexive: For *every* $a \in A$, $(a, a) \in R$ holds.
  - Note: Not applicable to $R \subseteq A \times B$ in general
  - Example: for $\{1, 2, 3\}$, $\{(1,1), (2,2), (2,3), (3,3)\}$
- Irreflexive: For *every* $a \in A$, $(a, a) \in R$ *never* holds. E.g., $\{(1,2), (2,1), (2,3)\}$
- Non-reflexive: Neither of the above

‘≤’, “sits next to”

Properties of Relation

For, e.g., structure classification

- Example relations
  - ‘<’
  - ‘≤’
  - “reports to” (corporate organization)
  - “sits next to” (people in class)
  - “sues” (people)
  - “is networked with” (computers)

Schematic representation

Symmetry

For a binary relation $R \subseteq A \times A$

- Symmetric: For *every* $(a, b) \in R$, $(b, a) \in R$ holds.
  - Example: $\{(1, 2), (2, 1), (2, 3), (3, 2), (3, 3)\}$
- Antisymmetric: If *both* $(a, b) \in R$ and $(b, a) \in R$ hold, $a = b$. [OK if either $(a, b) \in R$ or (b, a)]
  - E.g., '≤'
- Non-symmetric: Neither of the above
  - E.g., $\{(1, 2), (2, 3), (3, 2)\}$

"sits next to", '<', '≤'

Properties of Relation

For, e.g., structure classification

- Example relations
  - '<'
  - '≤'
  - "reports to" (corporate organization)
  - "sits next to" (people in class)
  - "sues" (people)
  - "is networked with" (computers)

Schematic representation

Transitivity

For a binary relation $R \subseteq A \times A$

- Transitive: For *every* $(a, b) \in R$ and $(b, c) \in R$, $(a, c) \in R$ holds.
  - Example: $\{(1, 2), (2, 3), (1, 3), (2, 2)\}$
- Non-transitive: Not transitive
  - Example: $\{(1, 2), (2, 3), (2, 2)\}$

'≤', "sits next to", "sues"

Section Summary

- Reflexivity
  - Reflexive
  - Irreflexive
  - Non-reflexive
- Symmetry
  - Symmetric
  - Antisymmetric
  - Non-symmetric
- Transitivity
  - Transitive
  - Non-transitive

Summary Exercise

- Discuss the connection between sets and relations.
  - Can we represent any set as a relation? [discussed today]
  - Can we represent any relation as a set? [Take-Home Exercise 3]
- Questions/Comments/Suggestions