

B1: Extra Exercises

- A. Draw a Venn diagram to analyze the general case of involving three sets and formally represent each region.

What about if four sets are involved?

- B. Justify: $(A \cap B)' = A' \cup B'$

- C. Describe the following in English:

$$(A \cap B \cap C)' = A' \cup B' \cup C'$$

For your practice

Foreign Policy

[Final Exam Practice, Spring 2003]

- Suppose that you are the foreign minister of a country whose foreign policy is that “an enemy of an enemy is a friend.” In a UN session, you must defend this policy.
- Then, you insist that every other country is “either with us or against us.”

What kind of tool should we use to convince others?

B2: Relations

Today

- Formally represent relations and use them to analyze phenomena
 - Formal representation of relation
 - Relation type
 - Basic properties of relation
- Take-home exercises
 - Professionals, Electronic Map

getting more abstract

Section 1

Relations in “Foreign Policy”

- “an enemy of an enemy is a friend”
 - “is a friend of”
 - “is an enemy of”
- “either with us or against us”
 - Explain this in terms of the connection between “is a friend of” and “is an enemy of”?
 - Explain this using the combined relation “is an enemy of an enemy of”?

Other Relation Examples

- Barbie dolls
- North pole
- Objects in a room
- Drug trafficking
- Professionals
- Corporate organization
- Computer files

General Properties of Relation

- Relation can be between **two/two or more** objects.
- Relation
 - Must be between objects from the same set/
 - Can be between objects from different sets.
- In general, the object ordering **matters/does not matter**.
- In general, one object **can/cannot** be related to multiple objects.

Justification/examples?

Formal Representation of Relation

For the case of relation between *two* objects

- Requirements: Must be able to
 - Combine two objects
 - Represent ordering between the objects
 - Include a number of object associations
 - Extend to relations between more than two objects

Use of sets?

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Ordered Pair

- A pair of two elements in a *specific order*
 - Notation: (a, b) or $\langle a, b \rangle$
 - Property: $(a, b) = (c, d) \Leftrightarrow a = c$ **and** $b = d$
- Examples
 - $doll_1$ wears $shirt_2 \Rightarrow (doll_1, shirt_2)$
 - l_2 sues $l_2 \Rightarrow (l_2, l_2)$
 - $file_{123}$ is in $folder_{234} \Rightarrow (file_{123}, folder_{234})$
- Note: $\{d_1, s_2\} = \{s_2, d_1\}$, but $(d_1, s_2) \neq (s_2, d_1)$

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Binary Relation

- A *set* of ordered pairs that characterize a certain association between two objects
- Example
 - $R = \{(d_1, s_2), (d_1, p_4), (d_2, s_3)\}$ Schematic representation
- Membership = The relation holds for the member (pair)
 - name actual definition
- Notation: $(d_1, s_2) \in R, d_1 R s_2, R(d_1, s_2)$

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Extension to Multiple Objects

- Extension of ordered pair
- Extension of binary relation

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(Ordered) n -Tuple

- Extension of ordered pairs to n elements
 - Notation: (a_1, a_2, \dots, a_n) or $\langle a_1, a_2, \dots, a_n \rangle$
 - Special cases: triple, quadruple, etc.
- Example: $(1, 2, 3) \neq (1, 3, 2) \neq (2, 1, 3)$, etc.

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n -ary Relation

- Extension of binary relation to n elements
 - Notation: $(a, b, c) \in R, R(a, b, c)$
- Special names
 - **Unary**: 1-ary relation (= set or 'property')
 - E.g., set of animals = unary relation on objects that have the "animal" property
 - Notation: $Animal(George)$
 - **Ternary**: 3-ary relation
 - E.g., "United 112 connects PHL and LAX"

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Section Summary

To represent a relation

- Connect two or more things \Rightarrow pair/ n -tuple
 - E.g., $(a, b) / (a_1, a_2, \dots, a_n)$
- Collect these \Rightarrow set of pairs/ n -tuples
 - E.g., $\{(d_1, s_2), (d_1, p_4), (d_2, s_3)\}, \{(a_1, a_2, \dots, a_n), \dots\}$

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Group Exercise

- Formally represent the reporting structure in your family as a relation.
- Formally represent the circle $x^2 + y^2 = 1$ as a relation. In addition, explain whether this relation can be a function. Let us define: \mathbf{R} = the set of real numbers.

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Section 2

Classifying Relation

- Relation involving objects from a **single** set
 - E.g., “is placed above”
- Relation involving objects from **different** sets
 - E.g., “file is contained in *directory*”

How to provide information about sets (from which objects are taken)?

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Not in Text!

Relation Type

- Relation involving objects from a single set
 - E.g., “is placed above”
 - I.e., $(a, b) \in P$ where $a \in O$ and $b \in O$
 - Type: $O \times O$
- Relation involving objects from different sets
 - E.g., “file is contained in *directory*”
 - I.e., $(x, y) \in C$ where $x \in F$ and $y \in D$
 - Type: $F \times D$

cf. data type in programming

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Cartesian Product

- Cartesian product of sets A and B :
 $\{(x, y) \mid x \in A \text{ and } y \in B\}$
 - Notation: $A \times B$
- Examples
 - $A = \{a, b\}, B = \{1, 2, 3\}$
 - $A \times A = \{(a, a), (a, b), (b, a), (b, b)\}$
 - $A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

2D plane, 3D space?

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Binary Relation Description

- Equivalent descriptions of binary relation R
 - Associates objects in set A and objects in set B
 - Has type $A \times B$
 - $R \subseteq A \times B$
- Example
 - $A = \{a, b\}, B = \{1, 2, 3\}$
 - $R = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

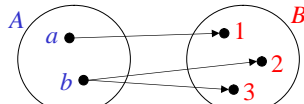
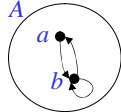
$A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

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Schematic Representation

- $A = \{a, b\}$, $B = \{1, 2, 3\}$
- $R \subseteq A \times A$
and $R = \{(a, b), (b, a), (b, b)\}$
- $S \subseteq A \times B$
and $S = \{(a, 1), (b, 2), (b, 3)\}$



- Identify the type
- Identify set members
- Use arrows

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Section Summary

- Type of a relation indicates the source of objects.
- The Cartesian product (e.g., $A \times B$) is used to indicate the type of a relation.

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Group Exercise

Recall “Foreign Policy”

- Define: $C = \{1, 2, 3, 4\}$
- Define: $friendOf = \{(1, 2), (2, 1), (2, 3), (3, 2)\}$
- Define: $enemyOf = ?$

1. Types?
2. Would it be reasonable to analyze $enemyOf$ as the complement of $friendOf$? Why?
3. Is an enemy of an enemy of a friend?

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Section 3

Properties of Relation

For, e.g., structure classification

- Example relations
 - $<$
 - \leq
 - “reports to” (corporate organization)
 - “sits next to” (people in class)
 - “sues” (people)
 - “is networked with” (computers)

Schematic representation

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Reflexivity

- For a relation $R \subseteq A \times A$
- **Reflexive:** For **every** $a \in A$, $(a, a) \in R$ holds.
 - Note: Not applicable to $R \subseteq A \times B$ in general
 - Example: for $\{1, 2, 3\}$, $\{(1,1), (2,2), (2,3), (3,3)\}$
- **Irreflexive:** For **every** $a \in A$, $(a, a) \in R$ **never** holds. E.g., $\{(1,2), (2,1), (2,3)\}$
- Non-reflexive: Neither of the above

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\leq , “sits next to”

Properties of Relation

For, e.g., structure classification

- Example relations
 - $<$
 - \leq
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Schematic representation

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Symmetry

For a **binary** relation $R \subseteq A \times A$

- **Symmetric:** For **every** $(a, b) \in R$, $(b, a) \in R$ holds.
 - Example: $\{(1, 2), (2, 1), (2, 3), (3, 2), (3, 3)\}$
- **Antisymmetric:** If **both** $(a, b) \in R$ and $(b, a) \in R$ hold, $a = b$. [OK if either $(a, b) \in R$ or (b, a)]
 - E.g., ' \leq '
- **Non-symmetric:** Neither of the above
 - E.g., $\{(1, 2), (2, 3), (3, 2)\}$ "sits next to", ' $<$ ', ' \leq '

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Properties of Relation

For, e.g., structure classification

- Example relations
 - ' $<$ '
 - ' \leq '
 - "reports to" (corporate organization)
 - "sits next to" (people in class)
 - "sues" (people)
 - "is networked with" (computers)

Schematic representation

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Transitivity

For a **binary** relation $R \subseteq A \times A$

- **Transitive:** For **every** $(a, b) \in R$ and $(b, c) \in R$, $(a, c) \in R$ holds.
 - Example: $\{(1, 2), (2, 3), (1, 3), (2, 2)\}$
- **Non-transitive:** Not transitive
 - Example: $\{(1, 2), (2, 3), (2, 2)\}$

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' \leq ', "sits next to", "sues"

Section Summary

- Reflexivity
 - Reflexive
 - Irreflexive
 - Non-reflexive
- Symmetry
 - Symmetric
 - Antisymmetric
 - Non-symmetric
- Transitivity
 - Transitive
 - Non-transitive

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Summary Exercise

- Discuss the connection between sets and relations.
 - Can we represent any set as a relation? [discussed today]
 - Can we represent any relation as a set? [Take-Home Exercise 3]
- Questions/Comments/Suggestions

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