Unit B3: Functions, 9/30/03

Exercise 1: Corporate Organization

Consider a small organization consisting of the following members:

- 1 CEO
- 2 managers
- 3 staff members

We know that this organization has a common sense reporting structure and that the CEO and both manager have at least one subordinate.

A. Give formal definitions of the following structure components that would be consistent with the information given above:

   - Set of people (give arbitrary names to distinguish them), *P*
   - Set of CEO's, *C*
   - Set of managers, *M*
   - Set of staff members, *S*
   - Relation "reports to", *R*

B. Would it be possible to define *R* as a function? If yes, give a formal definition (call it *reportsTo*). If no, explain.

C. Consider the *opposite* of "reports to", i.e., "supervises". (i) Can this be a relation? (ii) Can this be a function? Explain.

D. We know that the CEO earns at least $20,000, a manager, at least $30,000, and a staff member, at least $40,000. Assuming that their salary figures can be represented as a natural number, we also consider the set of natural numbers, **N**, as a part of the structure. To represent the salary information, we define a function, *salary*. Give the "type" of the function *salary* (**as discussed in class**). Then, define the function that would contain the complete information about the members' salaries (use arbitrary figures that are consistent with the given condition).

E. Does your function *salary* have an inverse? Would such a function *in general* have an inverse? Explain.

Exercise 2: Virtual Pet

In the behaviorist tradition of psychology, even human behavior was viewed in terms of a stimulus-response model. Although that tradition is no longer the mainstream in modern psychology, the idea is still useful to model, say, a virtual pet. In this exercise, we model our virtual pet, *LazyBoy*, as a creature that exhibits a simple stimulus-response pattern shown below (use abbreviation in your answer).

		Stimuli		
		Bison (*b*)	Geisha (*g*)	Wormhole (*w*)
States	Puzzled (*p*)	Puzzled (*p*)	Happy (*h*)	Mad (*m*)
	Happy (*h*)	Mad (*m*)	Puzzled (*p*)	Sleepy (*s*)
	Mad (*m*)	Happy (*h*)	Mad (*m*)	Puzzled (*p*)
	Sleepy (*s*)	Sleepy (*s*)		

For example, when *LazyBoy* is happy and see a bison, it becomes mad; when it is sleepy, regardless of the stimulus, it stays sleepy. This is an awfully simple model, but you could imagine that many aspects of publicly-available virtual pets and other games have some component built on this idea.

A. Give formal definitions of the following structure components that would characterize the virtual pet:

- Set of states, *States*
- Set of stimuli, *Stimuli*
- Function that characterizes the stimulus-response model of *LazyBoy*, called *behave*

Hint: The function *behave* is binary, i.e., with two inputs. Refer to lecture slides regarding how to define a binary function.

B. Give the “type” of the function *behave* (**as discussed in class**).

Note: Do not use any additional sets. Define the “type” of behave using only the available sets.

C. Identify whether the function *behave* is surjective, injective, or bijective. Justify your answer.

D. Does the function *behave* have an inverse? Explain.

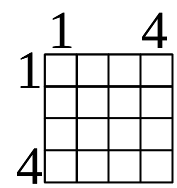
E. Suppose that the initial state is puzzled. In what state would *LazyBoy* be in after seeing a geisha and a wormhole in that order? Also represent the same state as a result of using the function twice from the given initial state.

Hint: Pay attention to what the function returns. Re-use the return of the function as the input of another use of the function.

F. [optional] The behavior of certain animals may not be represented as a function. Identify a potential problem with the use of a function for this purpose.

Note: Using complex forms of functions, we could represent a variety of animal behaviors reasonably well. For example, since we can define a function with an infinite members, at least theoretically, we could make it as precisely as we want. Since we can define an *n*-ary function with all sorts of conditions as input, we could characterize an arbitrarily complex behavior. We could even define a function that simulate probabilistic and nondeterministic behaviors (how?). Carefully review the general properties of functions and identify some property of animals that goes against them.

Exercise 3: Board Game



The board shown at right can be represented as follows:

- *Side* = {*i* | *i* is an integer *and* $1 \leq i \leq 4$}
- *Board* = *Side* × *Side*

Let us suppose that there is exactly one game piece but nothing else on the board. The piece can move on the board according to the following rule, applied in the specified order:

1. If there is a space above, the piece can move up.
2. If the above condition fails and there is a space to the right, the piece can move right.
3. If the above conditions fail and there is a space below, the piece can move down.
4. If the above conditions fail and there is a space to the left, the piece can move to the left.
5. If all of the above conditions fail, the piece stays.

Hint: With the above information, you can determine the next position for every board position.

We will represent the movement of the piece as function *m*.

A. Give the “type” of the function *m* (**as discussed in class**).

B. Give a formal definition of the function *m*. Use the list notation. Do not refer to any of the conditions.

C. Suppose that the piece is currently located at (1, 1). After 5 moves, where would the piece be located?

D. Suppose that the piece is currently located at (2, 4). Formally represent the position after 3 moves, as an instance of *function composition*. Do not directly state the position of *m*.

Hint: Pay attention to what the function returns. Can you re-use the return of the function?

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