Mini Project (Phase 1)

- Due at the beginning of Module B Evaluation Workshop.
- Choose an object/phenomenon *unique* to you.
- Informally describe the logic-structure connection involving the object/phenomenon.
- No length requirement. Must be word-processed (except diagrams).
- Self-evaluation: 1 pt if requirements satisfied.

Read project page

Review Questions

- Type of a relation? [E.g., B2 Ex1-C]
- How to justify properties of a relation?
  - Reflexive: For *every* $a \in A$, $(a, a) \in R$ holds.
  - Symmetric: For *every* $(a, b) \in R$, $(b, a) \in R$ holds.
  - Antisymmetric: If *both* $(a, b) \in R$ and $(b, a) \in R$ hold, $a = b$. [I.e., cannot have both unless $a = b$]
  - Transitive: For *every* $(a, b) \in R$ and $(b, c) \in R$, $(a, c) \in R$ holds.

B4: Structures

Today

- Formally represent structures and use them to analyze phenomena
  - Formally represent a structure
  - Overview of structure classification
  - Structures for language acquisition
- Take-home exercises
  - Human body, El Capitan

Section 1

Defining a Structure

- How to organize structure components to define a structure?
  - Multiple structure components
  - Helpful for comparing multiple structures

Structure

- An *n*-tuple involving sets, relations, and functions. Why not sets?
- Convention (in this course)
  - Structure name in Roman bold, e.g., **Structure**
- Example
  - **Max** = (**R**, *max*)
    R = the set of real numbers set
    max: $\mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$ function type
    $max = \{((x, y), z) \mid \text{if } x > y, z = x; \text{ otherwise } z = y\}$ function definition

Levels of Structure Definition

- Complete definition
  - All structure components are completely define.
  - E.g., **Max** in the previous slide
- Definition of a collection of structures
  - Some structure components show *only types* but no actual definitions.
  - E.g., **RealBinaryOp** = (**R**, *op*)
    R = the set of real numbers
    op: $\mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$ type, but *no* function definition
- **Max** is an instance of **RealBinaryOp**.

Section Summary

- Structure definition
  - **Name** = n-tuple
  - Structure component definition and types
- Degree of structure definition
  - Complete
  - Definition of a class

cf. Java abstract class/interface

Group Exercise

- Completely define a structure (**NorthPole**) to represent North Pole (as we discussed in class/exercises)
  - Set of Santa Claus, S
  - Set of reindeer, R
  - Set of objects, O (define this concisely!)
  - Relation "carries", *carries*

Include the "type" of relation

Section 2

Structure Classification

- Operational structures
  - Involves sets and functions, but no relations
  - E.g., **Max**, **PrimitiveCount**, **Strings**
- Relational structures
  - Involves sets and relations, but no functions
  - E.g., **Graph**, (**R**, ≤), **Professionals**
- Hybrid structures
  - Involves sets, relations, and functions

Fine-Tuned Classification

- Operational structures
  - Properties of function/operation: surjective, injective, bijective
  - Properties of binary function/operation: associative, identity element, commutativity
- Relational structures
  - Properties of relation: reflexivity, symmetry, transitivity

Section Summary

- Broad classification
  - Operational, relational, hybrid
- Fine-tuned classification
  - Based on properties of relation/function

Group Exercise

A. Define a structure (**Org**) to represent a corporate organization

Define completely with made-up information

B. Is **Org** an operational, relational, or hybrid structure?

C. What kind of properties does the relation/function possess?

Section 3

Modeling Language Acquisition

- Very rough language acquisition process
  - 12 months: objects (mommy, daddy, etc.)
  - 15 months: common nouns (toy, food, etc.)
  - 18 months: two-word sentences
  - 24 months: three-word sentences
  - After that

Representing these stages using structures?

12 Months: Objects

- $\mathbf{Lang_{12}}$ = (*Objects*)
- *Objects* = {i, m, d, f, g, t, u, v}

visual association

- i
- m
- d
- f
- g
- t

limitations?

15 Months: Common Nouns

- $\mathbf{Lang_{15}}$ = (*Objects*, *Food*, *Toy*, *Mad*)
- *Objects* = {i, m, d, f, g, t, u, v}
- *Food* = {f, g}
- *Toy* = {t, u, v}
- *Mad* = {d}

lexicon (dictionary)

semantics

- m
- f
- Food
- Toy
- Mad

Common nouns as set

18 Months: Two Words

- $\mathbf{Lang_{18}}$ = (*Objects*, *Food*, *Toy*, *Mad*)
- *Objects* = {i, m, d, f, g, t, u, v}
- *Food* = {f, g}
- *Toy* = {t, u, v}
- *Mad* = {d}

- m
- Food
- f is Food
- t is Toy
- d is Mad

statement (True/False)

English "is" as '∈'

syntax

24 Months: Three Words

- $\mathbf{Lang_{24}}$ = (*Objects*, *Food*, *Toy*, *Mad*, *PlayWith*)
- *Objects* = {i, m, d, f, g, t, u, v}
- *Food* = {f, g}
- *Toy* = {t, u, v}
- *Mad* = {d}
- *PlayWith* = {(i, t), (d, f)}

PlayWith: *Objects* ´ *Objects*

- m
- Food
- f (is) Food
- i PlayWith t
- d PlayWith f
- i ~~PlayWith~~ f

Verbs as binary relation

syntax even more crucial

After That

- Use of function?
- Adjectives?
- Relative clauses?
- Logical words: and, or, if, ...?
- Ambiguity?
- Understanding and generation?

B2 Ex3: Pair as a Set

Conditions

- $(a, b) \neq (b, a)$
- $(a, b) = (c, d) \Leftrightarrow (a = c \text{ and } b = d)$
- $(a, a) \neq (a) \neq a$

n-tuples in general ...

Textbook Ex. 19, p. 54

Summary Exercise

- Are you ready to do today's take-home exercises?
  - If yes, very briefly explain how you got all the necessary information, techniques, etc.?
  - If no, what do you need to do?
- Questions/Comments/Suggestions