Unit B5: Propositional Logic, 10/7/03

Exercise 1: System Description

Suppose that you are to deal with the following system description (each statement is given a specific propositional variable).

System description: If the system software is being installed, users cannot access the file system. If users can access the file system, then they can save new files. If users cannot save new files, then the system software is not being installed.

A. Identify propositions in the above system description and assign them single alphabetic characters. For example, you can assign *i* to "the system software is being installed."

B. Formally represent all the logical statements in the system description and list them with sequential numbers starting from 1 (as done in class).

   Hint: One sentence corresponds to one statement. But a statement may involve multiple "propositions" combined by "connectives."

C. Is the system description consistent? Justify, based on an assignment of truth values. That is, if you can find some truth assignments (for all the propositional variables) that would make all the statements true.

   Hint: Take advantage of the semantics of '$\rightarrow$'.

D. Connect all the statements listed in Question B using '$\wedge$'. Is the resulting statement a tautology? **Explain**.

E. In addition to the system description, also consider the following statement as part of the hypothesis for this problem: "the system software is being installed." Prove that "users can save new files" by filling in the following proof pattern (partially completed).

1. *i* [hyp] ⇐ The additional hypothesis
2. _______________ [hyp] ⇐ One of the available statements
3. _______________ [__________________] ⇐ See "Analysis Techniques" slides
4. _______________ [MP: 1, 3] ⇐ The conclusion

F. Repeat Question ~~D~~**E** using "proof by contradiction" using the following proof pattern.

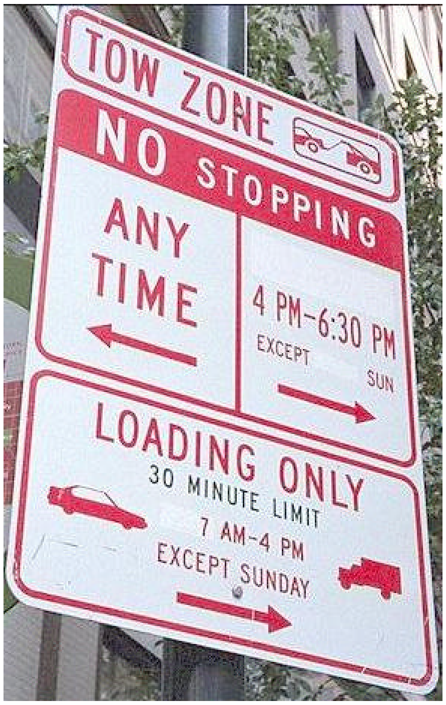
1. *i* [hyp]
2. | ________________ [hyp to be rejected]
3. | ________________ [hyp]
4. | ________________ [___________]
5. | **F** [contradiction: 1, 4]
6. s [proof by contradiction: 2-5]

G. In addition to the system description, also consider the following statement as part of the hypothesis for this problem: "users can***not*** save new files." Prove that "users can***not*** access the file system ***and*** the system software is ***not*** being installed."

Note: The hypothesis of this question is different from that of Questions E and F.

Exercise 2: *Streets of Philadelphia* (if you dare)

The streets of Philadelphia are a good place to boast your knowledge of logic. By now, you must be able to interpret the street sign at right as a logical statement shown below.



$$\big(left \rightarrow no\big) \wedge \big(right \rightarrow \big(((eve \wedge \neg sun) \rightarrow no) \wedge ((day \wedge \neg sun) \rightarrow load)\big)\big)$$

If you cannot make the statement true, you are violating the regulation. The meaning of the propositional variables used above are as follows:

- *left*: Parking to the left of the pole
- *right*: Parking to the right of the pole
- *no*: No stopping
- *day*: Between 7 a.m. and 4 p.m.
- *eve*: Between 4 p.m. and 6:30 p.m.
- *sun*: Sunday
- *load*: Loading only (30 minute limit)

Spending a lot of time looking at the sign, we notice that on Sunday, we can park to the right of the pole without problem. This suggests that the subexpression "$\big((eve \wedge \neg sun) \rightarrow no\big) \wedge \big((day \wedge \neg sun) \rightarrow load\big)$" can be transformed into an equivalent form "$sun \vee \big((eve \rightarrow no) \wedge (day \rightarrow load)\big)$".

A. As a preliminary step, justify that $x \vee (y \wedge z)$ is equivalent to $(x \vee y) \wedge (x \vee z)$. This is called "distributivity."

Note: You may analyze all possible truth value assignments (easier, but tedious) or prove it formally (more challenging).

B. Consider "$sun \vee \big((eve \rightarrow no) \wedge (day \rightarrow load)\big)$" as the hypothesis and prove "$\big((eve \wedge \neg sun) \rightarrow no\big) \wedge \big((day \wedge \neg sun) \rightarrow load\big)$."

Hint: See "Analysis Techniques" slides and also use "distributivity" justified in Question A above.

Exercise 3: Conditional '®'

First, review the truth table entry for '→'. In class, you were told to accept this as given. However, some of you may still wonder why. We will investigate this point.

Let us consider the following four possibilities, labeled #1 through #4. These possibilities will be discussed in the questions that follow.

p	q	$p \to q$			
		#1	#2	#3	#4
T	**T**	**T**	**T**	**T**	**T**
T	**F**	**F**	**F**	**F**	**F**
F	**T**	**T**	**T**	**F**	**F**
F	**F**	**T**	**F**	**T**	**F**

A. Some of these possibilities would be identical to the value of one of the propositional variable or some other connective. Analyze each case: #1, #2, #3, and #4. Would it make sense to use '→' if there is another way to represent it?

B. Although connectives '∧', '∨', and '↔' are all commutative, '→' is not. That is, the interpretations for $p \to q$ and $q \to p$ are not necessarily the same. For example, the following two sentences should not have the same truth conditions: (i) "if Joe is married, he is over 21", (ii) "if Joe is over 21, he is married". Which of the four possibilities (#1, #2, #3, or #4) is/are not commutative (i.e., candidates for '→')?

C. One type of reasoning we commonly use is "contrapositive." For example, in order to analyze "if Joe is married, he is over 21", we can equivalently analyze "if Joe is *not* over 21, he is *not* married". Which one of the four possibilities (#1, #2, #3, or #4) would make sense with respect to contrapositive?

D. Consider the following sentence: "all married people are over 21" (no formalization covered yet). Now, if nobody is married, we still consider the statement true (vacuously). Which interpretation (#1, #2, #3, or #4) would correspond to this analysis?

E. Considering all of the above, only one interpretation satisfies all the desirable properties. Which interpretation of '→' would you consider most appropriate for logical analysis?

<End>