Unit B6: First-Order Logic, 10/10/03

Sample solutions to these exercises will be posted on 10/13/03. But do these problems without seeing the solutions. Being able to understand provided solutions is *completely* different from being able to come up with a solution. Even an incomplete or incorrect answer of your own is better than understanding someone else' answer.

When you start working, you will face all sorts of real-life problems. Most of them are challenging, open-ended, and "ill-defined." You will never see a solution (if there were one, why would you be solving it?). In addition, even though you may not know the "correct" or "satisfactory" answer, you will still have to estimate where your answer stands (cf. self-evaluation in this course).

Exercise 1: Professionals

Represent in FOL all of the following statements from "Professionals."

- Everyone is mad.
- There is at least one doctor.
- There are at least two lawyers.
- Doctors are not lawyers.
- Lawyers sue everyone.
- Doctors sue back if they are sued.
- There is an individual who does not sue.

Here are sample basic expressions you can use in your FOL statements:

- x is mad: $mad(x)$
- x is a doctor: $doctor(x)$
- x is a lawyer: $lawyer(x)$
- x sues y: $sue(x, y)$

Note: All the individual variables must be *quantified*. That is, you should not write "Doctors are mad" as "$doctor(x) \rightarrow mad(x)$". Instead, write "$\forall x\ (doctor(x) \rightarrow mad(x))$" so that the variable x is quantified.

Answer:

1. $\forall x\ mad(x)$
2. $\exists x\ doctor(x)$
3. $\exists x\ \exists y\ (doctor(x) \wedge doctor(y) \wedge x \neq y)$
4. $\forall x\ (doctor(x) \rightarrow \neg lawyer(x))$
5. $\forall x\ \forall y\ (lawyer(x) \rightarrow sue(x, y))$
6. $\forall x\ ((doctor(x) \wedge \forall y\ (sue(y, x)) \rightarrow sue(x, y)))$ [another, equivalent form: $\forall x\ \forall y\ (doctor(x) \rightarrow (sue(y, x) \rightarrow sue(x, y)))$]
7. $\exists x\ \neg\exists y\ sue(x, y)$ [another, equivalent form: $\exists x\ \forall y\ \neg sue(x, y)$]

Exercise 2: Mystery Object

Here is a list of statements about some mystery object.

1. Every A-type object is attached to some object.
2. No A-type object can be attached to an A-type object.
3. An object rotates if no object is attached to it.
4. There is one B-type object with a flat top.
5. There are four pieces of A-type objects.

Here are sample basic expressions you can use in your FOL statements:

- x is an A-type object: $a(x)$
- x is a B-type object: $b(x)$
- x has/with a flat top: $flatTop(x)$
- x rotates: $rotate(x)$
- x is attached to y: $attachedTo(x, y)$

1. Represent Condition 1 in FOL.

   Hint: Consider the following statement equivalent to the one give above, "for any object x, there is some object y such that if x is an A-type object, then x is attached to y."

Answer: $\forall x\, \exists y\, (a(x) \rightarrow attachedTo(x, y))$

2. Represent Condition 2 in FOL.

   Hint: Formalize "some A-type object can be attached to an A-type object" and negate the whole statement to obtain Condition 2.

Answer: $\neg\exists x\, \exists y\, (a(x) \wedge a(y) \wedge attachedTo(x, y))$

3. Represent Condition 3 in FOL.

   Hint: "An object" here means "any object."

Answer: $\forall x\, ((\neg\exists y\, attachedTo(x, y)) \rightarrow rotate(x))$

4. Represent Condition 4 in FOL.

   Hint: "One B-type object" here means "exactly one B-type object." You may use two statements: one to represent "at least one B-type object" and another to represent "at most one B-type object." Note that two statements can always be joined to form a single statement using '$\wedge$'.

Answer:

- $\exists x\, (b(x) \wedge flatTop(x))$
- $\neg\exists x\, \exists y\, (b(x) \wedge b(y) \wedge x \neq y)$

5. Explain how you could represent Condition 5 in FOL. You do not need to show the actual statement as it can be very long.

   Hint: "Four pieces" here means "exactly four pieces." As in 4., you may use two statements.

Answer:

- $\exists u\, \exists v\, \exists w\, \exists x\, (a(u) \wedge a(v) \wedge a(w) \wedge a(x) \wedge u \neq v \wedge u \neq w \wedge u \neq x \wedge \ldots \wedge w \neq x)$
- $\neg$(statement that there are at least five A-type objects)

6. [optional] Were you able to guess what the object is?

Answer: The object I had in my mind was a skateboard.

<End>