Module B Evaluation Workshop

- Fri., Oct. 17, class time (one week from today)
- Review the relevant part of the syllabus and the on-line handbook
- Re-use your manila folder or large envelope (or prepare one)
- Complete and bring “Take-Home Exercise Self-Evaluation Form” (distributed today) along with exercises
- Exercise B6 will be returned that day. Include it in the pile then.
- Complete and bring “Module B Comprehensive Exercises” (available on-line)
- Group evaluation sessions (open book): 20 min × 3
- “Comprehensive Exercise Self-Evaluation Forms” will be distributed that day (no need to print in advance)
- Submit all materials *including mini project* at the end of the session

Two Types of Logic

- Propositional logic
  - Can express true/false statements and their combinations
  - Basic component of first-order logic
- First-order logic (FOL)
  - Can express “*every*” and “*some*”
  - Can specify structures

many more types of logic ...

B6: First-Order Logic (FOL)

Today

- Understand how to use FOL
  - Review & preview
  - Symbols/syntax
- Take-home exercises
  - Professionals, Mystery structure

Connection
- English-FOL
- Syntax-semantics
- Logic-structure

Section 1

Relations [B2]

- Reflexive: For *every* $a \in A$, $(a, a) \in R$ holds.
  " a Î A ((a, a) Î R)
- Irreflexive: For *every* $a \in A$, $(a, a) \in R$ *never* holds.
  " a Î A ((a, a) Ï R)
- Symmetric: For *every* $(a, b) \in R$, $(b, a) \in R$ holds.
  " a Î A " b Î B ((a, b) Î R ® (b, a) Î R)

every, any, All: "

Functions (1) [B3]

- Surjective: For *every* $b \in B$, there is *at least one* $a \in A$ such that $(a, b) \in f$.
  " b Î B $a Î A ((a, b) Î f)

at least one, some, a/an, Exists: $

Quantifier ordering is important!

Primitive Counting [A3]

- Associative: For *any* x, y, z, $(x + y) + z = x + (y + z)$
  " x " y " z ((x + y) + z = x + (y + z))
- Identity: There is *an* element x such that $x + y = y + x = y$ for *any* y.
  $x " y (x + y = y + x = y)

quantifier ordering, again

Omission of source set (if understood): " a Î A

Section Summary

- First-Order Logic (FOL) involves
  - Connectives as in propositional logic
  - Quantifiers
    - '∀' for "*for All*"
    - '∃' for "*there Exists*"
    - Variables that refer to *elements* in a set, e.g., ***x*** Î ***Set***
- FOL can express relations, e.g., *R*, '∈', '<'
- FOL can specify structures, e.g., Professionals

Section 2

First-Order Logic (FOL)

- Logic of individual elements
  - Cf. propositional logic = logic of statements
- Distinctive features
  - Can express relations/functions
  - Can deal with quantifiers: ∀ and ∃
  - Can specify structures

also called '(First-Order) Predicate Logic'

Symbols of FOL (1)

- Connectives: ¬, ∧, ∨, →, ↔
- Parentheses: (,)
- Individual constants: e.g., ***j***, ***m*** (could mean "*John*" and "*Mary*") real semantics later
- Predicate/relation symbols (*n*-ary): applies to *n* individuals, e.g., ***k*** (could mean "*kick*")
- Example relation symbol (syntax) vs. relation (semantics)
  - ***k*(*m*, *j*)** (could mean "*Mary kicks John*")

Symbols of FOL (2)

- Quantifiers: Universal '∀'; Existential '∃'
- Individual variables: e.g., ***x***, ***y*** (may range over individuals)
- Examples
  - **" *x* *k*(*m*, *x*)** "*Mary kicks everyone*"
  - **" *x* *k*(*x*, *j*)** "*Everyone kicks John*"
  - **" *x* ($*y* *k*(*x*, *y*))** "*Everyone kicks someone*"

cf. the other ordering

A single set involved
⇒ May abbreviate ∀*x* ∈ *A* (...) as ∀*x* (...)

Symbols of FOL (3)

- Function symbols (*n*-ary): e.g., ***s*** (could mean "spouse of")
- Equality symbol: '='

function symbol (syntax) vs. function (semantics)

- Examples
  - ***k*(*m*, *s*(*j*))**
  - ***k*(*s*(*m*), *s*(*j*))**
  - **$*x* (*x* = *s*(*m*))**
  - **" *x* ($*y* (*k*(*x*, *y*) ® *k*(*y*, *x*)))**
  - **$*x* (" *y* ((*x* = *s*(*m*)) ® *k*(*x*, *y*)))**

wff's and Terms

- wff: Corresponds to a proposition (true/false)
  - Diagnostics: Result of applying
    - A predicate: e.g., ***k*(*j*, *m*)**
    - '=': e.g., ***j* = *s*(*m*)**
    - Connectives: e.g., **(*j* = *s*(*m*)) Ù *k*(*j*, *m*)**
    - '∀' and '∃': e.g., **$*x* *k*(*j*, *x*)**
- Term: Corresponds to an individual (who)
  - Diagnostics:
    - Individual constant/variable: e.g., ***j***, ***x***
    - Result of applying a function: e.g., ***s*(*m*)**

Group Exercise

- Kicking Family
  - Constant symbols: ***j*** (“John”), ***m*** (“Mary”)
  - Binary predicate symbol: ***k*** (“kicks”)
  - Unary function symbol: ***s*** (“spouse of”)
- Express the following in FOL:
  a. “John is not Mary”
  b. “If John kicks Mary, Mary kicks John”
  c.“Everyone kicks everyone”
  d. “If John kicks someone, s/he is his spouse”

true/false?

Functions (2) [B3]

- Injective: For *every* $b \in B$, there is *at most one* $a \in A$ such that $(a, b) \in f$.

$\text{"}b\ \text{Î}\ B\ \text{Ø}(\text{\$}a_1\ \text{Î}\ A\ \text{\$}a_2\ \text{Î}\ A\ (a_1\ \text{¹}\ a_2\ \text{Ù}\ (a_1, b)\ \text{Î}\ f)\ \text{Ù}\ (a_2, b)\ \text{Î}\ f))$

at most one ⇔ not at least two
at least two: $\text{\$}a_1\ \text{\$}a_2\ (a_1\ \text{¹}\ a_2)$
at most one: $\text{Ø\$}a_1\ \text{\$}a_2\ (a_1\ \text{¹}\ a_2)$ ⇔ $\text{"}a_1\ \text{"}a_2\ (a_1 = a_2)$

- $\text{Ø"}x\ \text{j}\ \text{Û}\ \text{\$}x\ \text{Ø j}$
- $\text{Ø\$}x\ \text{j}\ \text{Û}\ \text{"}x\ \text{Ø j}$

Exercise

North Pole [A2]

- Reindeer *exists*.
- Reindeer are not Santa Claus.
- Reindeer must carry *someone/something*.
- Santa Claus must be carried by reindeer.

- ***reindeer***(_)
- ***santa***(_)
- ***carry***(_, _)

Exercise

Objects in a Room [Ex A1-1]

- An object must have another object on top of it.
- An object cannot be on top of itself.
- If an object X is on top of another object Y, Y cannot be on top of X.

onTop(_, _)

Preview

B6 Exercise 1: Professionals

- *Everyone* is mad.
- There is *at least one* doctor.
- There are *at least two* lawyers.
- Doctors are not lawyers.
- Lawyers sue *everyone*.
- Doctors sue back if they are sued.
- There is *an* individual who does not sue.

- ***mad***(_)
- ***doctor***(_)
- ***lawyer***(_)
- ***sue***(_, _)

Represent all the statements in FOL.

Exercise

Definition of Limit [Calculus]

$$\lim_{x \to a} f(x) = c$$

- For every real number $\varepsilon > 0$, there exists a real number $\delta > 0$ such that $|f(x) - c| < \varepsilon$ whenever $0 < |x - a| < \delta$.

Section Summary

- FOL combines terms to form wff's.
- FOL can represent a wide variety of conditions, which can be used to specify structures.
- In order to analyze the real meaning of wff's, we need to know the semantics of FOL.

Summary Exercise

- Depending on your previous experience with logic, this unit may have been difficult. Describe your situation.
- Question/Comments/Suggestions