Module B Evaluation Workshop

- Fri., Oct. 17, class time (one week from today)
- Review the relevant part of the syllabus and the on-line handbook
- Re-use your manila folder or large envelope (or prepare one)
- Complete and bring "Take-Home Exercise Self-Evaluation Form" (distributed today) along with exercises
- Exercise B6 will be returned that day. Include it in the pile then.
- Complete and bring "Module B Comprehensive Exercises" (available on-line)
- Group evaluation sessions (open book): 20 min × 3
- "Comprehensive Exercise Self-Evaluation Forms" will be distributed that day (no need to print in advance)
- Submit all materials *including mini project* at the end of the session

Two Types of Logic

- Propositional logic
  - Can express true/false statements and their combinations
  - Basic component of first-order logic
- First-order logic (FOL)
  - Can express "*every*" and "*some*"
  - Can specify structures

many more types of logic ...

B6: First-Order Logic (FOL)

Today

- Understand how to use FOL
  - Review & preview
  - Symbols/syntax
- Take-home exercises
  - Professionals, Mystery structure

Connection
- English-FOL
- Syntax-semantics
- Logic-structure

Section 1

Relations [B2]

- Reflexive: For *every* $a \in A$, $(a, a) \in R$ holds.
  $\forall a \in A ((a, a) \in R)$
- Irreflexive: For *every* $a \in A$, $(a, a) \in R$ *never* holds.
  $\forall a \in A ((a, a) \notin R)$
- Symmetric: For *every* $(a, b) \in R$, $(b, a) \in R$ holds.
  $\forall a \in A \forall b \in B ((a, b) \in R \rightarrow (b, a) \in R)$

every, any, All: $\forall$

Functions (1) [B3]

- Surjective: For *every* $b \in B$, there is *at least one* $a \in A$ such that $(a, b) \in f$.
  $\forall b \in B \exists a \in A ((a, b) \in f)$

at least one, some, a/an, Exists: $\exists$

Quantifier ordering is important!

Primitive Counting [A3]

- Associative: For *any* x, y, z, $(x + y) + z = x + (y + z)$
  $\forall x \forall y \forall z ((x + y) + z = x + (y + z))$
- Identity: There is *an* element x such that $x + y = y + x = y$ for *any* y.
  $\exists x \forall y (x + y = y + x = y)$

quantifier ordering, again

Omission of source set (if understood): $\forall a \in A$

Section Summary

- First-Order Logic (FOL) involves
  - Connectives as in propositional logic
  - Quantifiers
    - '∀' for "*for All*"
    - '∃' for "*there Exists*"
    - Variables that refer to *elements* in a set, e.g., $x \in Set$
- FOL can express relations, e.g., R, '$\in$', '$<$'
- FOL can specify structures, e.g., Professionals

Section 2

First-Order Logic (FOL)

- Logic of individual elements
  - Cf. propositional logic = logic of statements
- Distinctive features
  - Can express relations/functions
  - Can deal with quantifiers: ∀ and ∃
  - Can specify structures

also called '(First-Order) Predicate Logic'

Symbols of FOL (1)

- Connectives: $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$
- Parentheses: (,)
- Individual constants: e.g., j, m (could mean "*John*" and "*Mary*")

real semantics later

- Predicate/relation symbols (n-ary): applies to n individuals, e.g., k (could mean "*kick*")
- Example

relation symbol (syntax) vs. relation (semantics)

  - $k(m, j)$ (could mean "*Mary kicks John*")

Symbols of FOL (2)

- Quantifiers: Universal '∀'; Existential '∃'
- Individual variables: e.g., x, y (may range over individuals)
- Examples
  - $\forall x\, k(m, x)$ "*Mary kicks everyone*"
  - $\forall x\, k(x, j)$ "*Everyone kicks John*"
  - $\forall x\, (\exists y\, k(x, y))$ "*Everyone kicks someone*"

cf. the other ordering

A single set involved
⇒ May abbreviate $\forall x \in A$ (...) as $\forall x$ (...)

Symbols of FOL (3)

- Function symbols (n-ary): e.g., s (could mean "spouse of")
- Equality symbol: '='

function symbol (syntax) vs. function (semantics)

- Examples
  - $k(m, s(j))$
  - $k(s(m), s(j))$
  - $\exists x\, (x = s(m))$
  - $\forall x\, (\exists y\, ((k(x, y) \rightarrow k(y, x))))$
  - $\exists x\, (\forall y\, ((x = s(m)) \rightarrow k(x, y)))$

wff's and Terms

- wff: Corresponds to a proposition (true/false)
  - Diagnostics: Result of applying
    - A predicate: e.g., $k(j, m)$
    - '=': e.g., $j = s(m)$
    - Connectives: e.g., $(j = s(m)) \wedge k(j, m)$
    - '∀' and '∃': e.g., $\exists x\, k(j, x)$
- Term: Corresponds to an individual (who)
  - Diagnostics:
    - Individual constant/variable: e.g., j, x
    - Result of applying a function: e.g., $s(m)$

Group Exercise

- Kicking Family
  - Constant symbols: ***j*** ("John"), ***m*** ("Mary")
  - Binary predicate symbol: ***k*** ("kicks")
  - Unary function symbol: ***s*** ("spouse of")
- Express the following in FOL:
  a. "John is not Mary"
  b. "If John kicks Mary, Mary kicks John"
  c. "Everyone kicks everyone"
  d. "If John kicks someone, s/he is his spouse"

true/false?

Functions (2) [B3]

- Injective: For *every* $b \in B$, there is *at most one* $a \in A$ such that $(a, b) \in f$.

$\forall b \in B\ (\exists a_1 \in A\ \exists a_2 \in A\ (a_1 \neq a_2 \wedge (a_1, b) \in f \wedge (a_2, b) \in f))$

at most one ⇔ not at least two
at least two: $\exists a_1\ \exists a_2\ (a_1 \neq a_2)$
at most one: $\neg\exists a_1\ \exists a_2\ (a_1 \neq a_2) \Leftrightarrow \forall a_1\ \forall a_2\ (a_1 = a_2)$

- $\neg\forall x\ j \wedge \exists x\ \neg j$
- $\neg\exists x\ j \wedge \forall x\ \neg j$

Exercise

North Pole [A2]

- Reindeer *exists*.
- Reindeer are not Santa Claus.
- Reindeer must carry *someone/something*.
- Santa Claus must be carried by reindeer.

- ***reindeer***(_)
- ***santa***(_)
- ***carry***(_, _)

Exercise

Objects in a Room [Ex A1-1]

- An object must have another object on top of it.
- An object cannot be on top of itself.
- If an object X is on top of another object Y, Y cannot be on top of X.

onTop(_, _)

Preview

B6 Exercise 1: Professionals

- *Everyone* is mad.
- There is *at least one* doctor.
- There are *at least two* lawyers.
- Doctors are not lawyers.
- Lawyers sue *everyone*.
- Doctors sue back if they are sued.
- There is *an* individual who does not sue.

- ***mad***(_)
- ***doctor***(_)
- ***lawyer***(_)
- ***sue***(_, _)

Represent all the statements in FOL.

Exercise

Definition of Limit [Calculus]

$$\lim_{x \to a} f(x) = c$$

- For every real number $\varepsilon > 0$, there exists a real number $\delta > 0$ such that $|f(x) - c| < \varepsilon$ whenever $0 < |x - a| < \delta$.

Section Summary

- FOL combines terms to form wff's.
- FOL can represent a wide variety of conditions, which can be used to specify structures.
- In order to analyze the real meaning of wff's, we need to know the semantics of FOL.

Summary Exercise

- Depending on your previous experience with logic, this unit may have been difficult. Describe your situation.
- Question/Comments/Suggestions