Unit B6: Supplemental Notes, 10/10/03

Note: This applies only to Section 1 (i.e., the 9:30 a.m. class), in which I made a mistake in the exercise on North Pole [A2].

We discussed a FOL translation of "reindeer are not Santa Claus." I wrote the formula

(1) $\forall x\ \neg(\mathit{reindeer}(x) \rightarrow \mathit{santa}(x))$

However, it is not correct. I'm sorry. To analyze it further, let us use some properties of connectives (as can be found in the reference slides in B5).

(2)	1.	$\forall x\ \neg(\mathit{reindeer}(x) \rightarrow \mathit{santa}(x))$	[(1) above]
	2.	$\forall x\ \neg(\neg \mathit{reindeer}(x) \vee \mathit{santa}(x))$	['$\rightarrow$' shorthand: 1.]
	3.	$\forall x\ (\neg\neg \mathit{reindeer}(x) \wedge \neg \mathit{santa}(x))$	[De Morgan: 2.]
	4.	$\forall x\ (\mathit{reindeer}(x) \wedge \neg \mathit{santa}(x))$	[double negation: 3.]

That is, (1) is the same as (2) 4. But (2) 4. says that everyone is reindeer *and* not Santa Claus, which is incorrect.

The correct form must be as follows:

(3) $\forall x\ (\mathit{reindeer}(x) \rightarrow \neg \mathit{santa}(x))$

Note that we can apply '$\neg$' to a relation because relations are truth conditional, unlike individuals. Then, let us analyze this by using properties of propositional logic.

(4)	1.	$\forall x\ (\mathit{reindeer}(x) \rightarrow \neg \mathit{santa}(x))$	[(3) above]
	2.	$\forall x\ (\neg \mathit{reindeer}(x) \vee \neg \mathit{santa}(x))$	['$\rightarrow$' shorthand: 1.]
	3.	$\forall x\ \neg(\mathit{reindeer}(x) \wedge \mathit{santa}(x))$	[De Morgan: 2.]
	4.	$\neg\exists x\ (\mathit{reindeer}(x) \wedge \mathit{santa}(x))$	[crossing negation over quantifier (B6): 3.]

(4) 3. means that there is nothing that is both reindeer and Santa Claus.

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