

Unit C3: Ordered Structures, 10/31/03

Exercise 1: House Construction

In this exercise, you are the general contractor of a house construction project. A quick look at the Home Depot's *Outdoor Projects 1-2-3* revealed that you have to do various tasks in a specific order as shown below.

Conditions

- Foundation work (fd) must be done before framing (fr).
- Framing (fr) must be done before roofing (r).
- Roofing (r) must be done before exterior siding (es).
- Exterior siding (es) must be done before plumbing (p).
- Plumbing (p) must be done before flooring (fl).
- Plumbing (p) must be done before wall-board work (wb).
- Flooring (fl) must be done before carpeting (c).
- Flooring (fl) must be done before interior painting (ip).
- Exterior siding (es) must be done before wiring (wi).
- Wiring (wi) must be done before wall-board work (wb).
- Wall-board work (wb) must be done before interior painting (ip).
- Interior painting (ip) must be done before interior fixtures (if).
- Exterior siding (es) must be done before exterior painting (ep).
- Exterior painting (ep) must be done before exterior fixtures (ef).

- A. Draw a *Hasse diagram* that would show **Conditions** in a more visually appealing manner. You should use the abbreviation for each task.

We now represent the project as a structure **House** = (*Tasks*, <), where *Tasks* is the set of all the tasks involved in the conditions and '<' is a relation between two tasks with the meaning "must be done before" (this relation will be developed step by step).

- B. Tentatively, define the relation '<' as a set of pairs exactly as specified by **Conditions**, using the *list notation*. Note that **Conditions** are *not* explicit about the order between, say, foundation work (fd) and roofing (r). Thus, although you can infer such a sequence, do not include this type of information in '<'.

Note: The complete definition would be too long. Give only the first and last few elements.

- C. Show that this tentative definition of '<' is *not* transitive.

However, as hinted earlier, we can easily infer that foundation work (fd) must be done no later than roofing (r). In order to reflect this point *directly within the relation*, we will need to augment '<' with additional elements and make it transitive. During this process, we add only those elements that are needed to make the relation transitive, e.g., (fd, r). We do not want to destroy the ordering of the tasks; for example, we should not add (fr, fd) because we will no longer be able to tell the ordering of these two tasks. Let us call the new relation '<₂', which is transitive and still contains '<₁' as a subset. Then, '<₂' is called the **transitive closure** of '<'. As

