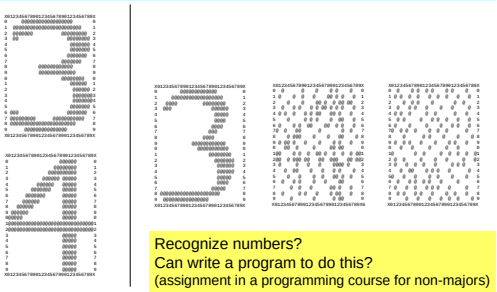
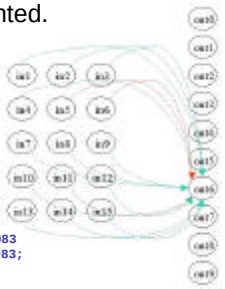
Character Recognition



Recognize numbers?
Can write a program to do this?
(assignment in a programming course for non-majors)

Artificial Neural Network (ANN)

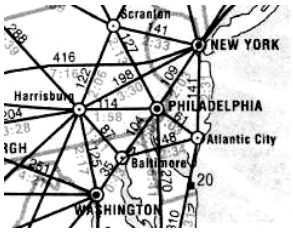
- Each connection is weighted.
- Contributes differently to the output



```
out6
= in1 * 0.083 + in2 * 0.083 + in3 * 0.083
+ in4 * 0.083 + in5 * -0.33 + in6 * -0.33
+ in7 * 0.083 + in8 * 0.083 + in9 * 0.083
+ in10 * 0.083 + in11 * -0.33 + in12 * 0.083
+ in13 * 0.083 + in14 * 0.083 + in15 * 0.083;
```


Electronic Map

- Representing a map as a structure



Example Problems

- Determine whether there is a connection between two points
- Find the shortest/fastest route between two points
- Find the shortest/fastest route connecting multiple points
- Detect whether you are looping around the same route

CMSC210: How to represent, *not* how to solve

Schematic Representation of Relations

- Relation $R = \{(a, b), (b, a), (b, c), (c, d)\}$

Hasse diagram?

C3: Graphs/Trees

Today

- Model connections of elements using graphs and trees
  - Graph and its variations as structure
  - Tree as a special case of graph
- Take-home exercises
  - Seating arrangements, Model your own

Section 1

City Streets

Structure?

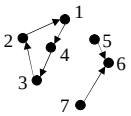
Digraph (Directed Graph)

- Definition: $\mathbf{G} = (V, E)$ where
  - V (vertices/nodes): a non-empty set representing points
  - E (edges): a binary relation on V representing connections

Merely a schematic representation of a relation?

$\mathbf{G_1}$

$\mathbf{G_2}$



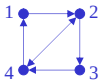
Digraph, Relation, Schematic

$\mathbf{G_1} = (\{1, 2, 3, 4\}, \{(1, 2), (2, 3), (3, 4), (4, 1), (2, 4), (4, 2)\})$

Digraph $\mathbf{G} = (V, E)$

Relation R ↔ Schematic representation

$R = \{(1, 2), (2, 3), (3, 4), (4, 1), (2, 4), (4, 2)\})$



Logical Specification

- Conditions for digraphs $\mathbf{G} = (V, E)$
  - $\exists x\ (x = x)$ specifies $V \neq \varnothing$
  - $\forall x\ (rel(x, x) \vee \neg rel(x, x))$ specifies $E: V \times V$
- Any structure satisfying the conditions is a digraph.

Logic-structure connection

Symmetric digraph?

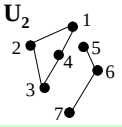
(Undirected) Graph

- $\mathbf{U} = (V, E)$ where
  - V: a non-empty set of vertices
  - E: a *symmetric* binary relation on V
- $\mathbf{U_1} = (V, E)$ where $V = \{a, b, c\}$, $E = \{(a, b), (b, a), (b, c), (c, b), (c, a), (a, c)\}$

Note: A graph is a special case of digraph. I.e., any graph is a digraph.

Hasse diagram?

$\mathbf{U_2}$



Schematic w/o arrows

Group Exercise 1

- An alternative definition of undirected graph as $\mathbf{U_{alt}} = (V, E)$ where
  - V: a non-empty set of vertices
  - E: a set of sets with cardinality 2

    E.g., $E = \{\{a, b\}, \{b, c\}, \{c, a\}\}$

Not a relation. Why?

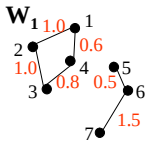
- Identify the pros and cons of this definition
  - Note: Also consider the connection to the definition of digraph.

(undirected) Weighted Graph

- **W** = (V, E, **R**, w) where
  - V: a non-empty set of vertices
  - E: a symmetric relation on V
  - **R**: the set of real numbers
  - w (weight): a function: $E \rightarrow$ **R**

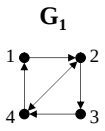
Analogous for digraph

Amida: Wanted to assign the height to a horizontal bar?

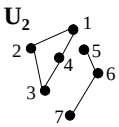


Connected Digraph/Graph

- Digraph/graph that is also connected
- Connected: From every vertex to any other one, there is a path.



Yes/No?



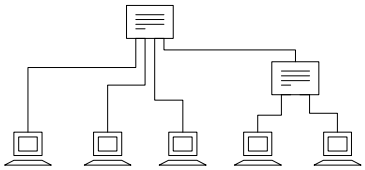
Yes/No?

Path

- A sequence of edges from x to y such that
  - Single-step: If $(x, y) \in E$, there is a path from x to y.
  - Multiple-step: If there is z such that $(x, z) \in E$ and there is a path from z to y, there is a path from x to y.

Recursive definition (more details later)

Network Connection

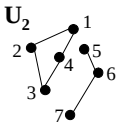


Do you know/remember IBM's "Token Ring" networks? What happened to them?

Acyclic (Undirected) Graph

- Graph with no cycles
- Cycle: A path from a vertex to itself *without repeating edges between the same vertices*

Note: The definition of cycle for digraphs slightly differ.



Section Summary

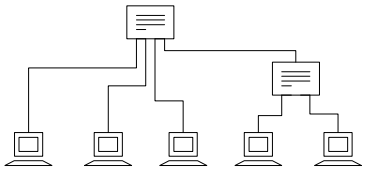
- Connections of points can be modeled as a graph, another type of structure.
- Graph variations
  - Digraph
  - Undirected graph
  - Weighted graph
  - Connected digraph/undirected graph
  - Acyclic digraph/undirected graph

Group Exercise 2

- For each of the following part of a human body:
  A. Cardiovascular system
  B. Digestive system
  C. Respiratory system
- Identify an appropriate type of graph with respect to the following:
  – Digraph or undirected graph
  – Connected or not
  – Acyclic or with cycles
- Give a simple schematic representation for each

Section 2

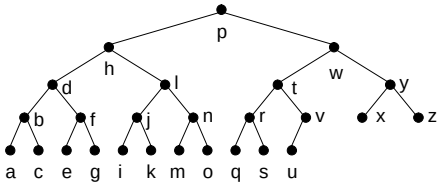
Ethernet Connection



Connected and acyclic

Example: Database

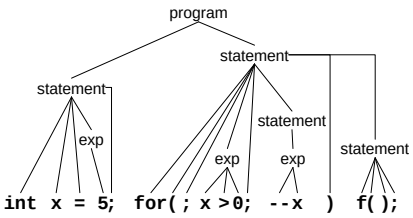
- Binary organization of indices



Other common properties?

Example: Compiler

- Structure of programming languages

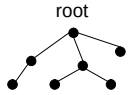


Tree

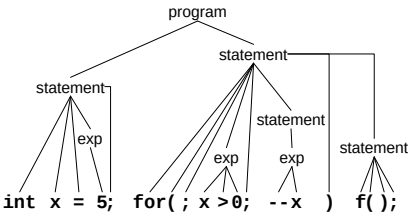
- Connected, acyclic, undirected graph with a designated root
- Examples
  – $\mathbf{T_1} = (V, E, root)$ where $V = \{a, b, c, d\}$, $E = \{(a, b), (b, c), (a, d)\}$, $root = a$

Note: In some case, the root is not included in the structure.

$\mathbf{T_2}$



Classification of Elements



Distinction between the bottom elements and others?

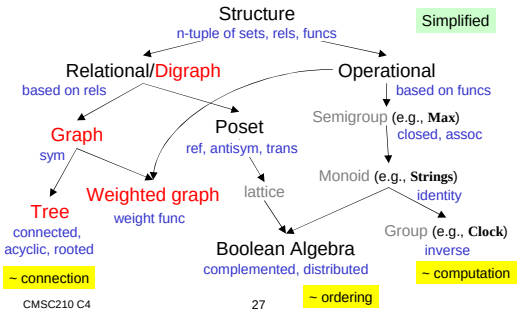
Distinguishing Nodes

- Leaf: a vertex with no children
- Child of v: u such that v is a parent of u
- Parent of v: an ancestor of v, u, such that $(u, v) \in E$
- Ancestor of v: u such that there is a path between v and the root through u
- Descendant of v: u such that v is an ancestor of u
- Height: the maximum number of nodes among the paths between a leaf and the root

Section Summary

- Tree is a special case of graph, appropriate for modeling a rooted, fan-out type hierarchy.

Organization of Structures



Group Exercise 3

Identify appropriate structures:

- Knowledge representation
- Sport tournament
- CS prerequisites
  - 210, 220, 230, 325, 330, 340, 350

Summary Exercise

- Do you think that studying various structures is useful for analyzing real-world phenomena? Concisely explain.
- Questions/Comments/Suggestions