

## Unit C6: Counting/Probability, 12/1/03

This is an additional exercise for practice. Sample solutions will be posted later.

### Exercise 1: Poker

Find the probability for each of the following hands in the game of Poker (consisting of 5 cards using the standard deck of 52 cards).

- Flush (e.g.,  $\spadesuit 2, \spadesuit 4, \spadesuit 5, \spadesuit 8, \spadesuit 9$ )
- Four cards (e.g.,  $\clubsuit 2, \heartsuit 2, \spadesuit 2, \heartsuit 2$ , *another*)
- Full house (e.g.,  $\clubsuit 2, \heartsuit 2, \spadesuit 7, \clubsuit 7$ )
- Straight (e.g.,  $\clubsuit A, \heartsuit 2, \spadesuit 3, \heartsuit 4, \clubsuit 5$ ; or  $\clubsuit 10, \heartsuit J, \spadesuit Q, \heartsuit K, \clubsuit A$ ; but *not*  $\clubsuit Q, \heartsuit K, \spadesuit A, \heartsuit 2, \clubsuit 1$ , i.e., A can be used as 1 or 13)
- Straight flush (e.g.,  $\heartsuit 2, \heartsuit 3, \heartsuit 4, \heartsuit 5, \heartsuit 6$ , again, A can be used as 1 or 13)
- Three cards (e.g.,  $\clubsuit 2, \heartsuit 2, \spadesuit 4, \clubsuit 7$ )
- Two pairs (e.g.,  $\clubsuit 2, \heartsuit 2, \spadesuit 5, \heartsuit 5, \clubsuit 8$ )
- One pair (e.g.,  $\clubsuit 2, \heartsuit 2, \spadesuit 4, \heartsuit 6, \clubsuit 7$ )

Note: For each hand, show how to compute the exact number of possibilities and give an approximate probability (1 to 2 significant digits).

Answer:

The information in [ ] is explanation, not part of the expression.

- Flush:  $C(4, 1)_{\text{[choose suit]}} \times C(13, 5)_{\text{[choose any 5 cards for the suit]}} = 5,148 \text{ ways} \Rightarrow 0.002$
- Four cards:  $C(13, 1)_{\text{[choose a \#]}} \times C(4, 4)_{\text{[choose all suits]}} \times C(12, 1)_{\text{[choose another \#]}} \times C(4, 1)_{\text{[choose a suit]}} = 624 \text{ ways} \Rightarrow 0.00024$
- Full house:  $C(13, 1)_{\text{[choose a \# (for 3 cards)]}} \times C(4, 3)_{\text{[choose 3 suits]}} \times C(12, 1)_{\text{[choose another \# (for a pair)]}} \times C(4, 2)_{\text{[choose 2 suits]}} = 3,744 \text{ ways} \Rightarrow 0.0014$
- Straight:  $C(10, 1)_{\text{[choose the starting \# between A and 10]}} \times C(4, 1)^5_{\text{[choose any suit for each of the 5 \#'s]}} = 10,240 \text{ ways} \Rightarrow 0.004$
- Straight flush:  $C(10, 1)_{\text{[choose the starting \# between A and 10]}} \times C(4, 1)_{\text{[choose a suit for the first \#]}} \times C(1, 1)^4_{\text{[the suits of the other 4 cards is fixed]}} = 40 \text{ ways} \Rightarrow 0.00014$
- Three cards:  $C(13, 1)_{\text{[choose a \#]}} \times C(4, 3)_{\text{[choose a suit]}} \times C(12, 2)_{\text{[choose two other \#'s]}} \times C(4, 1)^2_{\text{[choose suits for the 2 \#'s]}} \Rightarrow 0.02$   
Note: It is not correct to use  $C(12, 1) \times C(11, 1)$  instead of  $C(12, 2)$  because  $C(12, 1) \times C(11, 1) = P(12, 2) \neq C(12, 2)$
- Two pairs:  $C(13, 2)_{\text{[choose two \#'s]}} \times C(4, 2)^2_{\text{[choose suits for the 2 \#'s]}} \times C(11, 1)_{\text{[choose another \#]}} \times C(4, 1)_{\text{[choose a suit]}} \Rightarrow 0.05$
- One pair:  $C(13, 1)_{\text{[choose a \#]}} \times C(4, 2)_{\text{[choose 2 suits]}} \times C(12, 3)_{\text{[choose 3 other \#'s]}} \times C(4, 1)^3_{\text{[choose suits for the 3 \#'s]}} \Rightarrow 0.4$

<End>