Unit C6: Counting/Probability, 12/1/03

This is an additional exercise for practice. Sample solutions will be posted later.

Exercise 1: Poker

Find the probability for each of the following hands in the game of Poker (consisting of 5 cards using the standard deck of 52 cards).

- Flush (e.g., ♦2, ♦4, ♦5, ♦8, ♦9)
- Four cards (e.g., ♣2, ♦2, ♥2, ♠2, *another*)
- Full house (e.g., ♣2, ♦2, ♥2, ♠7, ♣7)
- Straight (e.g., ♣A, ♦2, ♥3, ♠4, ♣5; *or* ♣10, ♦J, ♥Q, ♠K, ♣A; but *not* ♣Q, ♦K, ♥A, ♠2, ♣1, i.e., A can be used as 1 or 13)
- Straight flush (e.g., ♥2, ♥3, ♥4, ♥5, ♥6, again, A can be used as 1 or 13)
- Three cards (e.g., ♣2, ♦2, ♥2, ♠4, ♣7)
- Two pairs (e.g., ♣2, ♦2, ♥5, ♠5, ♣8)
- One pair (e.g., ♣2, ♦2, ♥4, ♠6, ♣7)

Note: For each hand, show how to compute the exact number of possibilities and give an approximate probability (1 to 2 significant digits).
Answer:
The information in [ ] is explanation, not part of the expression.

- Flush: $C(4, 1)_{\text{[choose suit]}} \times C(13, 5)_{\text{[choose any 5 cards for the suit]}}$ = 5,148 ways ⇒ 0.002
- Four cards: $C(13, 1)_{\text{[choose a \#]}} \times C(4, 4)_{\text{[choose all suits]}} \times C(12, 1)_{\text{[choose another \#]}} \times C(4, 1)_{\text{[chose a suit]}}$ = 624 ways ⇒ 0.00024
- Full house: $C(13, 1)_{\text{[chose a \# (for 3 cards)]}} \times C(4, 3)_{\text{[choose 3 suits]}} \times C(12, 1)_{\text{[choose another \# (for a pair)]}} \times C(4, 2)_{\text{[choose 2 suits]}}$ = 3,744 ways ⇒ 0.0014
- Straight: $C(10, 1)_{\text{[choose the starting \# between A and 10]}} \times C(4, 1)^5_{\text{[chose any suit for each of the 5 \#'s]}}$ =10,240 ways ⇒ 0.004
- Straight flush: $C(10, 1)_{\text{[choose the starting \# between A and 10]}} \times C(4, 1)_{\text{[choose a suit for the first \#]}} \times C(1, 1)^4_{\text{[the suits of the other 4 cards is fixed]}}$ = 40 ways ⇒ 0.00014
- Three cards: $C(13, 1)_{\text{[choose a \#]}} \times C(4, 3)_{\text{[choose a suit]}} \times C(12, 2)_{\text{[choose two other \#'s]}} \times C(4, 1)^2_{\text{[choose suits for the 2 \#'s]}}$ ⇒ 0.02
  Note: It is not correct to use $C(12, 1) \times C(11, 1)$ instead of $C(12, 2)$ because $C(12, 1) \times C(11, 1) = P(12, 2) \neq C(12, 2)$
- Two pairs: $C(13, 2)_{\text{[choose two \#'s]}} \times C(4, 2)^2_{\text{[choose suits for the 2 \#'s]}} \times C(11, 1)_{\text{[choose another \#]}} \times C(4, 1)_{\text{[choose a suit]}}$ ⇒ 0.05
- One pair: $C(13, 1)_{\text{[choose a \#]}} \times C(4, 2)_{\text{[choose 2 suits]}} \times C(12, 3)_{\text{[choose 3 other \#'s]}} \times C(4, 1)^3_{\text{[choose suits for the 3 \#'s]}}$ ⇒ 0.4

<End>