

## Unit D1: Sets, 11/18/03

### Exercise 1: Web Pages

- A. Inductively define exactly the set  $W$  of all the web pages that are linked directly or indirectly from the TCNJ home page.

Consider a structure **Links** =  $(W, \rightarrow)$  where  $W$  is as defined in Question A above and ' $\rightarrow$ ' is a relation of the type  $W \times W$ , where " $x \rightarrow y$ " means "page  $x$  has a direct link to page  $y$ ."

- B. Analyze whether **Links** is one or more of the following: poset, digraph, undirected graph, tree. Note that you must carefully analyze the properties of ' $\rightarrow$ '.
- C. Suppose that a web search robot starts collecting all the pages linked from the TCNJ home page. What kind of potential problem the robot might encounter? Again, refer to the properties of ' $\rightarrow$ '.

### Exercise 2: Foreign Policy

Suppose that you are the foreign minister of a country whose foreign policy is that "an enemy of an enemy is a friend." In a UN session, you must defend this policy. Here, consider two *symmetric* relations *enemyOf* and *friendOf*, both, of the type  $E \times E$ , where  $E$  is the set of entities (e.g., individuals/countries). For example, *friendOf*(*Sam*, *François*) means "Sam is a friend of François."

First, you insist that every other country is "either with us or against us." A more general form of this position can be stated as follows. **Condition 1:** *enemyOf* is the complement of *friendOf*. You tried to argue for your policy based on this condition, but most other countries are not convinced.

- A. Analyze why most other countries are not convinced, *referring to relevant concepts in discrete math* (i.e., not referring to your belief, religion, philosophy, etc.).

Next, you drop **Condition 1** and consider the following alternative. **Condition 2:** *enemyOf* is intransitive, i.e.,  $\forall x \forall y \forall z ((\text{enemyOf}(x, y) \wedge \text{enemyOf}(y, z)) \rightarrow \neg \text{enemyOf}(x, z))$ . You tried to argue for your policy based on **Condition 2**, but most other countries are still not convinced.

- B. Analyze why most other countries are not convinced, *referring to concepts in discrete math*.

As a third attempt, you combine **Conditions 1** and 2.

- C. Would it be possible to support your foreign policy? **Explain**, *referring to concepts in discrete math*.

It became clear that other countries would not accept **Conditions** 1 and 2 for the first place. So, instead of characterizing friends based on *enemyOf*, you now try to define exactly the set  $F$  of friends recursively (assuming that *friendOf* is transitive).

D. Inductively define the set  $F$ .

You must have ~~excluded~~ included an “exclusion clause.” However, imagine that your partner in an evaluation workshop forgot it. Of course, without it, some unexpected result may surface.

E. Explain why the lack of exclusion clause would fail to define the set  $F$  correctly.

**Hint:** Consider  $Hans \notin F$  and his friends.

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