

## Unit D2 Exercise: Supplemental Notes, 12/1/03

### Exercise 2

B.

- (ii) The paradoxical statement can be represented in FOL. The key is that it is possible to define the notion of “this” in FOL. The idea is as follows. By properly coding the symbols used in FOL, it becomes possible to label all the FOL statements with sequential numbers. That is, all the FOL statements can be ordered linearly (imagine a dictionary of FOL statements). In addition, it is also possible to refer to natural numbers in FOL. Then, we can imagine a situation where statement  $i$  expresses that “statement  $i$  is false,” corresponding to “this sentence is false.”
- (iii) One of the consequences of (ii) is that there are FOL statements that cannot be proven true or false (undecidability). We can also show that certain true statements cannot be proven (incompleteness). The latter finding by Kurt Gödel (around 1930) was a huge blow to the mathematics community. The idea certainly stop the attempt of Bertrand Russel to formalize all of mathematics in FOL. To summarize, there is a limit to FOL and mathematics like everything else. One of the greatest discovery in math is that they realized this point. It’s probably the same for us regarding other things. Meta-cognition of understanding self-limitation will be a powerful tool for us to advance cautiously.

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