

Unit B7: Logic and Structure, 3/18/03

Due at the beginning of the next class meeting

Exercise H1: Spring Break

The bad news is that your spring break is over. The good news is that you can turn your spring break into a more memorable *form* (i.e., formal representation). First, consider the following symbols:

- Unary predicate/relation symbols: *leisure*, *academic*
- Binary predicate/relation symbol: *moreFun*

Let us suppose that the following logical statements would specify your spring break:

1. $\exists x \exists y (leisure(x) \wedge leisure(y) \wedge x \neq y)$
2. $\exists x academic(x)$
3. $\forall x (leisure(x) \vee academic(x))$
4. $\forall x \forall y (((leisure(x) \wedge academic(y)) \rightarrow moreFun(x, y))$
5. $\forall x \forall y ((leisure(x) \wedge leisure(y)) \rightarrow \neg(moreFun(x, y) \vee moreFun(y, x)))$

Consider a structure **SpringBreak** = (*Activities*, *Leisure*, *Academic*, *MoreFun*) where

- *Activities* contains all sorts of activities, e.g., leisure and academic activities.
 - *Leisure* and *Academic* are subsets of *Activities*, and interpret the unary predicate symbols *leisure* and *academic*, respectively. For example, *leisure(a)* is true if an activity *a* is a member of the set *Leisure*.
 - *MoreFun* defines the meaning of the predicate symbol *moreFun*. E.g., *moreFun(a, b)* is true if $(a, b) \in moreFun$, i.e., you have more fun with *a* than with *b*.
- A. Translate Condition 5 to a statement in plain English. Do not give a literal translation of logical symbols. Make your statement as easy as possible even to folks not in this class (of course, with exactly the same meaning).
- B. Could there be activities other than leisure or academic? **Explain.**
- C. Condition 3 alone does not exclude (potentially counterintuitive) possibilities of (1) $Academic \subseteq Leisure$ and (2) $Leisure \subseteq Academic$, because logical connective ‘ \vee ’ is not exclusive. Fully analyze both Case 1 and Case 2 with respect to their possibilities, by referring to relevant logical statement(s).
- D. Formally define all the structure components of the *smallest* instance of **SpringBreak** that would satisfy all the conditions listed above. For relations/functions, **give their types** as well.

Note: The smallest structure must contain the least possible number of elements in set(s), relation(s), and function(s), where applicable.

Exercise H2: North Pole (out of season)

In this problem, we will consider a hypothetical, out-of-season or off-duty north pole establishment that can be specified by logical statements involving the following symbols:

- Unary predicate/relation symbols: *santa*, *reindeer*
- Binary predicate/relation symbol: *feed*

Here are the logical statements:

1. $\neg\exists x (\neg\text{reindeer}(x) \wedge \neg\text{santa}(x))$
2. $\exists x \text{santa}(x)$
3. $\forall x\exists y \text{feed}(y, x)$
4. $\forall x \neg\text{feed}(x, x)$
5. $\forall x (\text{santa}(x) \rightarrow \neg\exists y \text{feed}(x, y))$
6. $\forall x\forall y (\text{feed}(x, y) \rightarrow \neg\text{feed}(y, x))$

Consider a structure **NorthPole** = (*Universe*, *Santa*, *Reindeer*, *Feed*) where

- *Universe* contains all sorts of elements, e.g., possibly Santa Claus and reindeer.
 - *Santa* and *Reindeer* are subsets of *Universe*, and interpret the unary predicate symbols *santa* and *reindeer*, respectively. For example, *reindeer*(*a*) is true if an object *a* is a member of set *Reindeer*.
 - *Feed* defines the meaning of the predicate symbol *feed*, possibly applicable to both Santa Claus and reindeer. For example, *feed*(*a*, *b*) is true if $(a, b) \in \text{feed}$, i.e., *a* feeds *b*.
- A. Transform Condition 1 so that there would be no negation (\neg) involved in the statement. Then, explain Condition 1 based on the transformed form. **Hint: Review Section 2 of Lecture B6 slides.**
 - B. Explain whether there must be at least one reindeer. Refer to relevant condition(s).
 - C. Explain whether there must be at least two reindeer. Refer to relevant condition(s).
 - D. Explain whether there must be at least three reindeer. Refer to relevant condition(s).
 - E. Formally define all the structure components of the *smallest* instance of **NorthPole** that would satisfy all the conditions listed above. For relations/functions, **give their types** as well.

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