

Questions/Review

- Ex A4
 - Part 1: TM
 - Binary addition
 - Your own
 - Part 2
 - Equivalence with an alternative version
 - Beyond TM

General form of memory? Subroutine?

CSC460 A5

1

Unit A5: Overview

- Discuss problems of the day
- Introduce “Computability” properties used to classify problems with respect to TM behavior **Intuition first; more precise discussion in Module B**
- Discuss the role of proof in this course
- Preview Module A Evaluation Workshop
- Preview Exercise A5 Module A Comprehensive Exercise

CSC460 A5

2

Problems du Jour

- Vending machine
- Compilation
- Termination detection (“**halting problem**”)
- Infinite-loop detection
- Weather forecast
- Jackpot prediction
- Driving

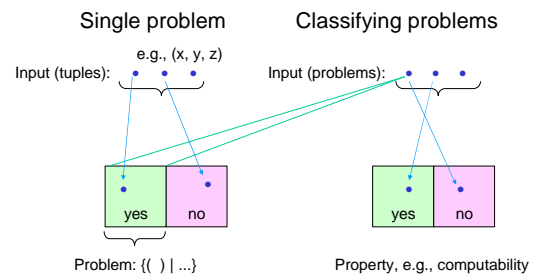
set representations?

Properties
 1. Always terminate?
 2. Impossible to solve?

CSC460 A5

3

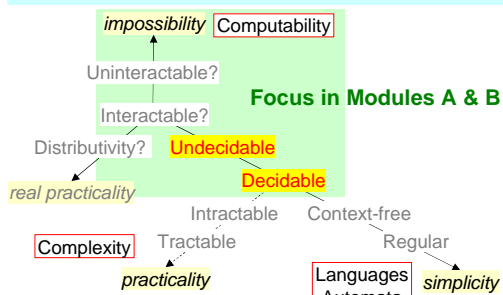
Problems and Properties



CSC460 A5

4

Overview: Theory of Computation



CSC460 A5

5

Decidable Problems

- Property
 - Some TM always terminates and either accepts or dies (rejects).
- Examples
 - Binary number addition
 - Palindrome detection
 - Vending machine
 - Compilation

CSC460 A5

6

Decidability Synonyms

- **Decidable** [of a procedure, problem]
- TM/algorithm exists, i.e., always terminates
- TM-decidable [of a TM]
- Computable [of a procedure/function]
- Solvable [of a problem]
- **Recursive** [of a set], cf. recursive function

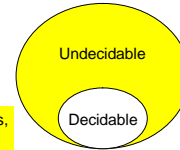
CSC460 A5

7

Undecidable Problems

- Property
 - **Not** decidable (*complement* of decidable problems)
 - Equivalently: Not always terminate, or not always end up in either accept state or die
- Examples
 - Halting problem
 - Floor tiling

Members are problems,
which are already sets



CSC460 A5

8

Undecidability Synonyms

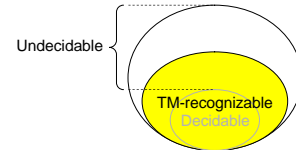
- **Undecidable**
 - Unsolvable
- **Note:** Incorrect definition of “unsolvable” on the earlier version of “Topics” doc. Please fix. I apologize.

CSC460 A5

9

TM-Recognizable Problems

- Property
 - Some TM can identify all the acceptable inputs
 - Note: OK if loops on non-acceptable inputs
- Example
 - All of decidable problems
 - Halting problem (undecidable)



CSC460 A5

10

TM-Recognizability Synonyms

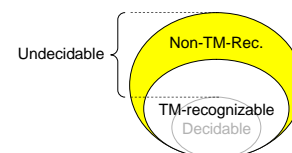
- **TM-recognizable**
- TM exists (but not necessarily always terminates)
- **Recursively enumerable (RE)** [of a set]

CSC460 A5

11

Non-TM-Recognizable Problems

- Property
 - No TM can represent
- Example
 - Infinite loop detection
 - Weather



CSC460 A5

12

Non-TM-Recognizability Synonym

- Non-TM-Recognizable (non-TM-rec.)
- Non-RE

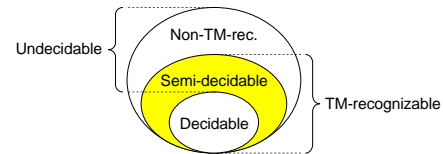
CSC460 A5

13

Semi-decidable Problems

Equivalent terms

- Semi-decidable
- TM-recognizable but not decidable



CSC460 A5

14

Property of “Co-”

- **co-X**
 - Consider the **complement** of a problem
 - If the complement has property X, the original problem is called **co-X**.
- **Examples**
 - Infinite-loop detection is co-TM-recognizable. I.e., the complement problem (halting problem) is TM-recognizable.
 - Palindrome detection is co-decidable.

CSC460 A5

15

Useful Theorems

- **Theorem:** Decidable \Leftrightarrow TM-recognizable *and* co-TM-recognizable
 - Note: Theorem as a statement in a theory, which needs to be proved
- **Corollary** (a theorem easily derivable from another theorem): The complement of a decidable problem is decidable.

CSC460 A5

16

Review: Proofs

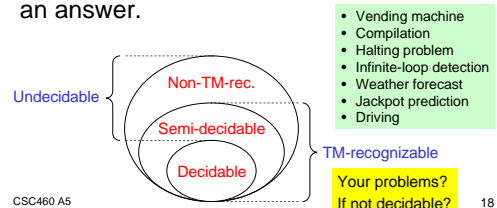
- **Proof** [Text 1.2-1.4]
 - A sequence of justifiable steps of deriving a theorem from axioms and other theorems using rules of inference
- **Example proof technique**
 - To prove a statement: $X \Leftrightarrow Y$
 - (1) Prove $X \Rightarrow Y$, *and* (2) Prove $X \Leftarrow Y$

CSC460 A5

17

Unit Summary

- With respect to impossibility, problems can be classified into 3 disjoint sets, based on whether a TM exists and/or always gives an answer.



CSC460 A5

18

Module A Evaluation Workshop

- Required materials (**hardcopy**) **print in advance**
 - Module A evaluation form
 - Supporting notes **Significance of supp. notes?**
 - Exercises
- Activities
 - Module review exercise
 - Peer discussion
 - Reflection and self-evaluation

Summary Question

- Did you get the big picture of the “computability” part of the Theory of Computation? Explain.
- Questions/Comments/Suggestions (make sure to understand the materials and also the evaluation procedure)