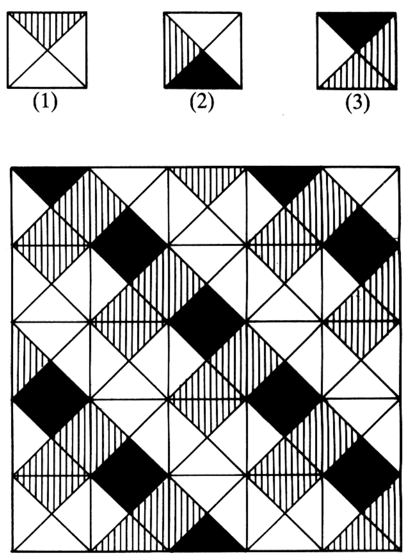
Name: ______________________

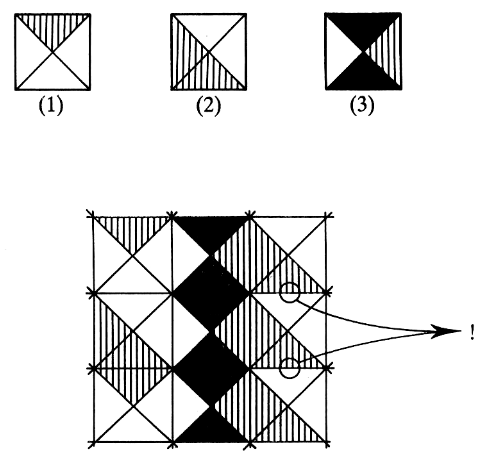
Module A Review Exercise, 2/8/05

Do <u>one</u> of the following problems. Write your response on back or on another sheet of paper.
Three computability properties to be used: decidable, semi-decidable, non-TM-recognizable

Option 1: Floor Tiling (Sample Problem 3)

Recall the “floor tiling problem.” That is, you will be using *n* types of *square* tiles with distinct patterns to fill the space assigned to you, which could be in any shape. You need to match the edges of the tiles as shown in the left example below (in this case, using three types of tiles). If you arrange your tiles as in the right example below, you will be considered “esthetically-challenged.” [Images from *Algorithmics* by David Harel]





In class, we briefly noted that it would ***not*** be possible to write a program to tell <u>whether or not</u> a given set of *n* types of square tiles can fill your floor with the above-mentioned condition (note: take this information as given). **Classify** this problem with respect to the three computability properties listed above. **Explain**.

Option 2: Arithmetic System (Sample Problem 7)

One of the great achievements in logic is that it can formalize a wide variety of systems. For example, the mathematical system of arithmetic can be formalized in first-order logic in a consistent (roughly, meaningful) way. Its basic components include axioms such as “for any x, $x + 0 = x$” and “for any x, $x \times 0 = 0$”. In this system, we can write all sorts of formulas that are representative of arithmetic. One property that a good logic system must have is that all “true” formulas can be proven from axioms using rules of inference. But it has been shown that ***not all*** true formulas in such an arithmetic system can be proven (note: take this information as given). **Classify** this problem with respect to the three computability properties listed above. **Explain**.

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