Name: ______________________

Exercise B2, 2/15/05

Part 1: Diagonalization

Today, we discussed that for countably-many TMs, there are uncountably-many languages. Thus, no matter how hard we try, there won't be enough TMs to cover all of the languages. This immediately suggests that the acceptability problem, i.e., $\{(M, i) \mid M \text{ accepts } i\}$, is undecidable. Among the many missing points in the uncountable set, we also identified a non-TM-recognizable set (problem), the "diagonalization language." Regardless of the nature of a problem, once represented as a set, the notion of countability offers an insight into analyzing computability classes. In this respect, understanding diagonalization is important. We will practice this technique along with *proof by contradiction* with a few more examples.

Task 1: **Power set of natural numbers**: Show that the power set of the set of natural numbers is uncountable, using diagonalization.

Task 2: **Languages**: We discussed that the set of all languages is uncountable relying on the fact that power set increases the set cardinality (i.e., the power set of a countable set is uncountable). For this task, show (the same result) that the set of all languages is uncountable, directly using diagonalization.

Part 2: Review "Discrete Math"

If necessary, review the following concepts in Discrete Math:

- Set cardinality
- Power set
- Set complement
- Countable
- Uncountable
- Proof by contradiction

Survey: Time spent between classes: ______________

// End