Name: ______________________

Module B Extra Problems, 3/4/05

This is an *optional* problem set. These are provided for those who want to try some textbook-style problems. The problems are actually taken from a classic textbook. If you want to discuss these problems, contact the instructor.

X1. Consider a model of a Turing machine in which each move permits the read-write head to travel more than one cell to the left or right, the distance and direction of travel being one of the arguments of δ. Give a precise definition of such an automata and sketch a simulation of it by a standard Turing machine. [Note: δ is the transition function.]

X2. Show that the following problem is undecidable. Given any Turing machine M, $a \in \Gamma$, and $w \in \Sigma^+$, determine whether or not the symbol a is ever written when M is applied to w. [Note: "Σ^+" is a regular expression, i.e., a sequence of one or more symbols in the set Σ.]

X3. Show that the following problems are undecidable:
A. $L(M)$ contains any string of length five.
B. $L(M)$ is regular.
[Note: $L(M)$ denotes the language accepted by a TM M.]

X4. Show that there is no algorithm for deciding if any two Turing machines M_1 and M_2 accept the same language. How would your conclusion be affected if M_2 is a finite automaton?

X5. Let B be the set of all Turing machines that halt when started with a blank tape. Show that this set is recursively enumerable, but not recursive. [Note: Review the terms.]

X6. Let M_1 and M_2 be arbitrary Turing machines. Show that the problem "$L(M_1) \subseteq L(M_2)$" is undecidable.

X7. Determine whether or not the following statement is true: Any problem whose domain is finite is decidable.

X8 [**Post's Correspondence Problem** (**PCP**)]. Consider a non-empty set of pairs of strings $S = \{(x_1, y_1), (x_2, y_2), ..., (x_k, y_k)\}$. The problem is to find some sequence of members of S such that the concatenation of the x strings are identical to that of y strings. $\Pi x_i = \Pi y_i$ where Π denotes the concatenation of strings and i denotes the index of a string. Show that PCP is undecidable. [Note: Do web search for more details about this well-known problem.]

// End