

Name: \_\_\_\_\_

## Exercise C4, 3/25/05

### Part 1: Review: First-Order Logic (FOL)

The Pumping Lemma involves a complicated interaction of first-order quantifiers ( $\forall$  and  $\exists$ ). In order to understand how to use it, we need to know the basics of FOL.

**Task** (optional): Review FOL, if necessary.

### Part 2: Game-Theoretic Interpretation of FOL

One way to interpret complex first-order statements is to use the game-theoretic interpretation.

**Task 1:** Write up Unit C4 Group Exercise 1.

**Task 2** (optional): ‘ $O$ ’ (asymptotic upper bound) is defined as follows:  $f(n) \in O(g(n))$  if there are constants  $c > 0$  and  $n_0 \geq 1$  such that  $f(n) \leq c g(n)$  for every integer  $n \geq n_0$ . Use the game-theoretic interpretation to analyze this.

**Task 3** (optional): The limit  $c$  of a function  $f(x)$  as  $x$  approaches  $a$ ,  $\lim_{x \rightarrow a} f(x) = c$ , is defined as follows: For every real number  $\varepsilon > 0$ , there exists a real number  $\delta > 0$  such that  $|f(x) - c| < \varepsilon$  whenever  $0 < |x - a| < \delta$ . Use the game-theoretic interpretation to analyze this.

### Part 3: Distinguishing Regular vs. Non-Regular Languages

Using the Pumping Lemma, we can prove that a certain language is *not* regular in a precise manner. For such languages, we will need to use a more powerful mechanism, e.g., PDA, TM.

**Task 1:** Write up Unit C4 Group Exercise 2.

**Task 2:** Redo the two problems (tasks) in Exercise C3 Part 2 using the tools discussed in Unit C4. Note that you will need to use different approaches to show that a language is regular or non-regular.

Survey: Time spent between classes: \_\_\_\_\_

Unit C4 slides for the in-class group exercises and take-home exercises.

Game-Theoretic Interpretation of FOL

- $\forall x \exists y$  (x kicks y) cf. logic-structure connection
    - **Falsifier** (tries to crack  $\forall$ ): Choose  $x$
    - **Verifier** (tries to support  $\exists$ ): Choose  $y$ , based on  $x$
    - Check whether "x kicks y" is true
  - $\exists x \forall y$  (x kicks y)
    - **Verifier**: Choose  $x$
    - **Falsifier**: Choose  $y$ , based on  $x$
    - Check whether "x kicks y" is true
- To satisfy: Play a **verifier**  
To reject: Play a **falsifier**

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Pumping Lemma (for regular languages)

To show that a language is **not** regular

- For **any** infinite regular language  $L$ ,
- there **exists** a positive integer  $n_0$  such that
- for **any**  $w \in L$  such that  $|w| \geq n_0$ ,
- there **exists** a decomposition  $w = xyz$  where  $|xy| \leq n_0$  and  $|y| \geq 1$  such that
- for **any**  $i \geq 0$ ,
- $xy^i z \in L$

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Logic Game for Pumping Lemma

Between a **verifier** (for  $\exists$ ) and a **falsifier** (for  $\forall$ )

- **Falsifier**: Choose an infinite language  $L$
- **Verifier**: Choose a positive integer  $n_0$  such that
- **Falsifier**: Choose  $w \in L$  such that  $|w| \geq n_0$ ,
- **Verifier**: Choose a decomposition  $w = xyz$  where  $|xy| \leq n_0$  and  $|y| \geq 1$  such that Meaning of  $|xy| \leq n_0$
- **Falsifier**: Choose  $i \geq 0$ ,
- Check whether  $xy^i z \in L$

To satisfy: Play a **verifier** (wrong)  
To reject: Play a **falsifier** (correct)

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Using Pumping Lemma

Play a **falsifier**: This is how this lemma is used.

- To show that  $0^n 1^n$  is **not** regular
- Anticipate any  $n_0 \geq 1$
- Choose  $0^n 1^n \in L$  such that  $2n \geq n_0$
- Anticipate any decomposition  $w = xyz$ :
  - Case 1:  $0^b 0^b 1^n$  ( $b \geq 1$ )
  - Case 2:  $0^n 1^c 1^d$  ( $c \geq 1$ )
  - Case 3:  $0^a (0^b 1^c) 1^d$  ( $b + c \geq 1$ )
- Choose  $i = 2$
- $0^n 0^{2b} 1^n \notin L$ ,  $0^n 1^{2c+d} \notin L$ , and  $0^a (0^b 1^c)^2 1^d \notin L$

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