Name: ______________________

Exercise C4, 3/25/05

Part 1: Review: First-Order Logic (FOL)

The Pumping Lemma involves a complicated interaction of first-order quantifiers ($\forall$ and $\exists$). In order to understand how to use it, we need to know the basics of FOL.

Task (optional): Review FOL, if necessary.

Part 2: Game-Theoretic Interpretation of FOL

One way to interpret complex first-order statements is to use the game-theoretic interpretation.

Task 1: Write up Unit C4 Group Exercise 1.

Task 2 (optional): '*O*' (asymptotic upper bound) is defined as follows: $f(n) \in O(g(n))$ if there are constants $c > 0$ and $n_0 \geq 1$ such that $f(n) \leq c\, g(n)$ for every integer $n \geq n_0$. Use the game-theoretic interpretation to analyze this.

Task 3 (optional): The limit c of a function $f(x)$ as x approaches a, $\lim_{x \to a} f(x) = c$, is defined as follows: For every real number $\varepsilon > 0$, there exists a real number $\delta > 0$ such that $|f(x) - c| < \varepsilon$ whenever $0 < |x - a| < \delta$. Use the game-theoretic interpretation to analyze this.

Part 3: Distinguishing Regular vs. Non-Regular Languages

Using the Pumping Lemma, we can prove that a certain language is *not* regular in a precise manner. For such languages, we will need to use a more powerful mechanism, e.g., PDA, TM.

Task 1: Write up Unit C4 Group Exercise 2.

Task 2: Redo the two problems (tasks) in Exercise C3 Part 2 using the tools discussed in Unit C4. Note that you will need to use different approaches to show that a language is regular or non-regular.

Survey: Time spent between classes: ______________

Unit C4 slides for the in-class group exercises and take-home exercies.

Game-Theoretic Interpretation of FOL

- $\forall x \exists y$ (x kicks y) cf. logic-structure connection
  - Falsifier (tries to crack $\forall$): Choose x
  - Verifier (tries to support $\exists$): Choose y, based on x
  - Check whether "x kicks y" is true
- $\exists x \forall y$ (x kicks y)
  - Verifier: Choose x
  - Falsifier: Choose y, based on x
  - Check whether "x kicks y" is true

To satisfy: Play a verifier
To reject: Play a falsifier

CSC460 C4 7

Pumping Lemma (for regular languages)

To show that a language is *not* regular

- For any <u>infinite</u> regular language L,
- there exists a positive integer n_0 such that
- for any $w \in L$ such that $|w| \geq n_0$,
- there exists a decomposition $w = xyz$ where $|xy| \leq n_0$ and $|y| \geq 1$ such that
- for any $i \geq 0$,
- $xy^iz \in L$

CSC460 C4 6

Logic Game for Pumping Lemma

Between a verifier (for $\exists$) and a falsifier (for $\forall$)

- Falsifier: Choose an infinite language L
- Verifier: Choose a positive integer n_0 such that
- Falsifier: Choose $w \in L$ such that $|w| \geq n_0$,
- Verifier: Choose a decomposition $w = xyz$ where $|xy| \leq n_0$ and $|y| \geq 1$ such that
- Falsifier: Choose $i \geq 0$,
- Check whether $xy^iz \in L$

Meaning of $|xy| \leq n_0$

To satisfy: Play a verifier (wrong)
To reject: Play a falsifier (correct)

CSC460 C4 10

Using Pumping Lemma

Play a falsifier: This is how this lemma is used.

- To show that 0^n1^n is *not* regular
- Anticipate any $n_0 \geq 1$
- Choose $0^n1^n \in L$ such that $2n \geq n_0$
- Anticipate any decomposition $w = xyz$:
  - Case 1: $0^a\ 0^b\ 1^n$ ($b \geq 1$)
  - Case 2: $0^n\ 1^c\ 1^d$ ($c \geq 1$)
  - Case 3: $0^a\ (0^b1^c)\ 1^d$ (b + c ≥ 1)
- Choose $i = 2$
- $0^a0^{2b}1^n \notin L$, $0^n1^{2c}1^d \notin L$, and $0^a(0^b1^c)^21^d \notin L$

CSC460 C4 12

// End