Mini Research

- Questions?
- Status?

Practicum evaluation
- Wed., Apr. 27
- Most of you are assigned earlier presentations 12:00pm ~
- Detailed instructions will be distributed later (but already available on-line)

Unit D3 (cont'd)

Today's Questions

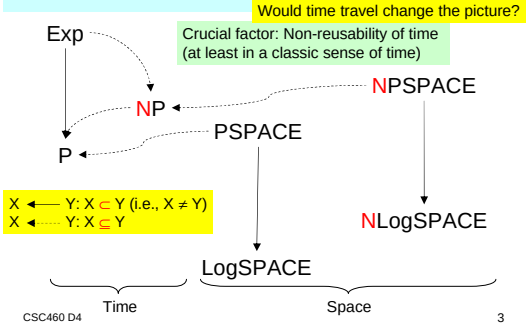
1. *X* vs. *X*SPACE
   - E.g., P vs. PSPACE, NP vs. NPSPACE
2. *X*SPACE vs. N*X*SPACE
   - E.g., PSPACE vs. NPSPACE, LogSPACE vs. NLogSPACE
3. *X* vs. N*Log(X)*SPACE
   - E.g., Exp vs. NPSPACE, P vs. NLogSPACE

Rev. 1

Complexity Hierarchy

Would time travel change the picture?

Crucial factor: Non-reusability of time (at least in a classic sense of time)



X ← Y: X ⊂ Y (i.e., X ≠ Y)
X ← Y: X ⊆ Y

Group Exercise: Question 2

- *X*SPACE vs. N*X*SPACE
  - E.g., PSPACE vs. NPSPACE, LogSPACE vs. NLogSPACE
- Which would be correct? Why?

A. *X*SPACE ⊆ N*X*SPACE
B. *X*SPACE = N*X*SPACE
C. *X*SPACE ⊇ N*X*SPACE
D. None of the above

Effects of Nondeterminism

Observation so far
- L(TM) = L(NTM)
- L(DFA) = L(NFA)
- L(DPDA) ⊆ L(PDA) PDAs are nondeterministic
- P ⊆ NP
  - Probably P ⊂ NP (i.e., P ≠ NP)

Savitch's Theorem

SPACE($f(n)$): Space-bounded by $O(f(n))$

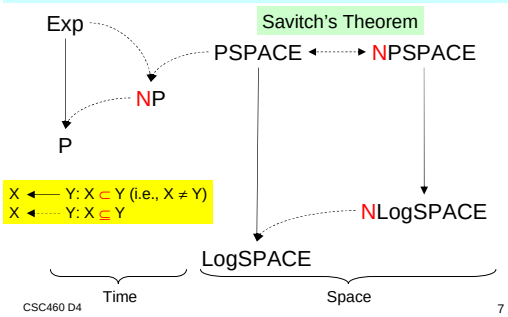
- Given a function $f(n) \geq n$,

  NSPACE($f(n)$) ⊆ SPACE($f^2(n)$)
  - Corollary: NPSPACE = PSPACE
- Proof idea
  - Elimination of exponential factor: Simulate nondeterminism using a stack à la "iterative deepening"
    May still take more than P time.
  - Source of the square: Multiplication by the stack height

Why the requirement: $f(n) \geq n$?

Rev. 2

Complexity Hierarchy



Group Exercise: Question 3

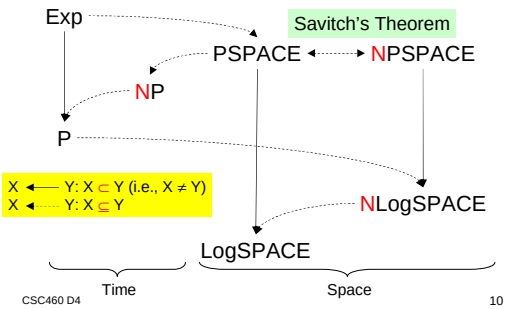
- *X* vs. N*Log(X)*SPACE
  - E.g., Exp vs. (N)PSPACE, P vs. NLogSPACE
- Which would be correct? Why?

A. $X \subseteq$ N*Log(X)*SPACE
B. $X =$ N*Log(X)*SPACE
C. $X \supseteq$ N*Log(X)*SPACE
D. None of the above

$X \supseteq$ N*Log(X)*SPACE

- Exp $\supseteq$ PSPACE idea
  - A TM using n space can have at most $n \times 2^{O(n)}$ different configurations (~ states, head position, tape symbols).
  - PSPACE would use Exp time.
- P $\supseteq$ NLogSPACE idea
  - MAZE is NLogSPACE-complete (i.e., can simulate any NLogSPACE problem).
  - Analogous to the above case

Complexity Hierarchy



D3 Unit Summary

- Time and space complexities have some connection.
- The effect of nondeterminism differ in each case.
- Applications of complexity theory
  - NPC and beyond $\Rightarrow$ Use of approximation algorithms
  - Effect of limited resources (of different types)

Unit D4: Overview

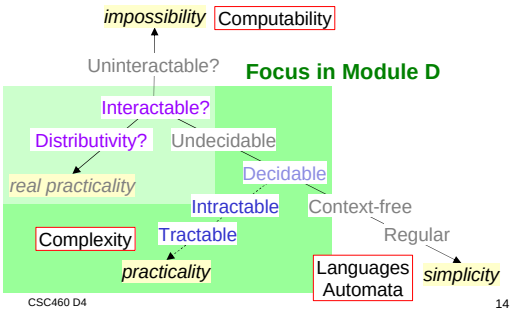
- Analyze the effects of parallel processing on theoretical aspects
- Explore examples of parallel processing
- Preview Exercise D4 "Reading: Critical View about TMs"

Parallelism as the ultimate dimension in computing?

Parallel Processing

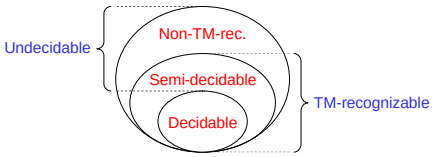
- Classification
  - A few processors in a single computer
  - Parallel machine
  - Distributed computing [multiple units/locations]
- Questions
  - Decide "undecidable" problems?
  - Move down Chomsky hierarchy?
  - Reduce time complexity (asymptotically)?

Overview: Theory of Computation



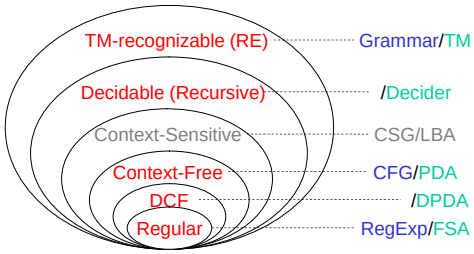
Review

Computability Summary



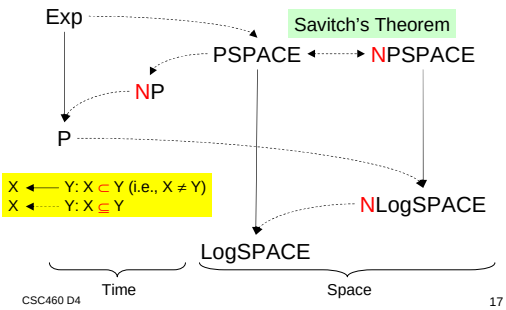
Review

Extended Chomsky Hierarchy



Review

Complexity Summary



Group Exercise 1

A. Choose at least two different types of parallel processing examples
B. Could they decide "undecidable" problems?
C. Could they move down the Chomsky hierarchy?
D. Could they reduce time complexity?

Classification of Tasks

- Instruction
  - Parallelizable vs. inherently sequential
- Data
  - Disjoint vs. shared

Classification of Parallelism

- SISD: Single Instruction Single Data
- SIMD: Single Instruction Multiple Data
- MISD: Multiple Instruction Single Data
- MIMD: Multiple Instruction Multiple Data

Flynn [1966]

Degree of Parallelism

May consider different types

- Constant, i.e., a fixed number k of parallelism
- Function of the input size, n
  - Assumption: A large number of processors are available so that given any reasonable n, every c input units can be assigned a processor (for linear case, other functions possible).

X Inst. Y Data
- SISD
- SIMD
- MISD
- MIMD

Massively parallel processors

Group Exercise 2

- Analyze whether the following cases of parallelism would affect: (a) computability, (b) complexity, (c) Chomsky hierarchy.

Case 1: Constant factor

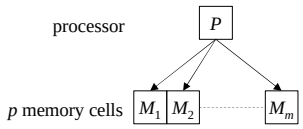
Case 2: Function of the input size, n

- Assumption: A large number of processors are available so that given any reasonable n, every c input units can be assigned a processor (for linear case, other functions possible).

Modeling parallelism?

RAM

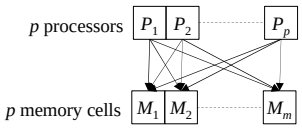
- Random Access Machine



- Processing cycle: read-compute-write
- Computation type: arithmetic
- Cf. URM, which is more primitive

PRAM

- Parallel Random Access Machine



- Processing cycle: read-compute-write in sync
- Processors run the same program, possibly different interpretation based on their index

Models of Parallel Processing

- PRAM
- Hypercube (2^d processors for d dimensions)
- Bounded Degree Network (at most d connections per processor)

Example Problems

- Search
- Finding the max
- Summation
- Matrix multiplication $c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj} \quad \text{for } 1 \le i, j \le n$
- Merging
- Merge sort

Handling conflicts
[C: common, E: exclusive
- EREW [not realistic]
- CREW
- CRCW
  - Common write
  - Priority write

Group Exercise 3

- Find the time complexity and processor complexity (number of processors) for the following problems:

1. Summation
2. Matrix multiplication $c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj} \quad \text{for } 1 \le i, j \le n$
3. Merge sort

Class NC "Nick's class"

- Informally, efficiently parallelizable class of problems
- Definition: Solvable on a PRAM in log time with polynomially-many processors
- NC $\subseteq$ LogSPACE $\subseteq$ P

Meaning of this?
Example of P problem not in NC?

Unit Summary

- Parallel processing can affect complexity, and Chomsky hierarchy, but not computability.
  - Some (not all) common problems have exponential speed up with parallel processing.
- The complexity of distributed computing in general is more difficult to analysis.

Summary Question

- If parallel processing does not affect the computability results, what would do so? Speculate.
- Questions/Comments/Suggestions